



Chapitre I: Rappels de Thermodynamique et Mécanique des Fluides

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Introduction

THIS chapter summarises the basic physical laws of fluid mechanics and thermodynamics, developing them into a form suitable for the study of turbomachines. Following this, some of the more important and commonly used expressions for the efficiency of compression and expansion flow processes are given.

The laws discussed are:

- (1) the *continuity of flow equation*;
- (2) the *first law of thermodynamics* and the *steady flow energy equation*;
- (3) the *momentum equation*;
- (4) the *second law of thermodynamics*.

All of these laws are usually covered in first-year university engineering and technology courses, so only the briefest discussion and analysis is give here. It is worth remembering that these laws are completely general; they are independent of the nature of the fluid or whether the fluid is compressible or incompressible.

The equation of continuity

Consider the flow of a fluid with density ρ , through the element of area dA , during the time interval dt . Referring to Figure 2.1, if c is the stream velocity the elementary mass is $dm = \rho \cdot c dt dA \cdot \cos\theta$, where θ is the angle subtended by the normal of the area element to the stream direction. The velocity component perpendicular to the area dA is $c_n = c \cos\theta$ and so $dm = \rho c_n dA dt$. The elementary rate of mass flow is therefore

$$\dot{m} = \frac{dm}{dt} = \rho c_n dA. \quad (1.1)$$

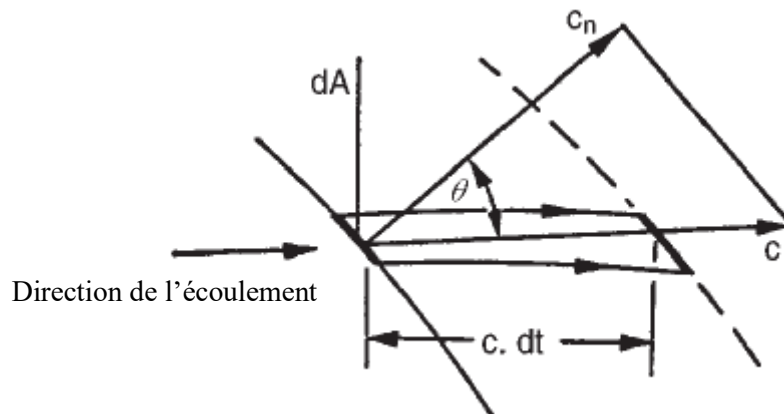


Figure 1.1. Ecoulement à travers un élément de surface.

Most analyses in this book are limited to one-dimensional steady flows where the velocity and density are regarded as constant across each section of a duct or passage. If A_1 and A_2 are the flow areas at stations 1 and 2 along a passage respectively, then

$$\dot{m} = \rho_1 c_{n1} A_1 = \rho_2 c_{n2} A_2 = \rho c_n A \quad (1.2)$$

since there is no accumulation of fluid within the control volume.

The first law of thermodynamics internal energy

The *first law of thermodynamics* states that if a system is taken through a complete cycle during which heat is supplied and work is done, then

$$\oint (dQ - dW) = 0 \quad (1.3)$$

Where $\oint dQ$ represents the heat supplied to the system during the cycle and $\oint dW$ the work done by the system during the cycle. The units of heat and work in eqn. (1.3) are taken to be the same. During a change of state from 1 to 2, there is a change in the property internal energy,

$$E_2 - E_1 = \int_1^2 (dQ - dW) \quad (1.4)$$

For an infinitesimal change of state

$$dE = dQ - dW \quad (1.4a)$$

The steady flow energy equation

Many textbooks, e.g. Çengel and Boles (1994), demonstrate how the first law of thermodynamics is applied to the steady flow of fluid through a control volume so that the steady flow energy equation is obtained. It is unprofitable to reproduce this proof here and only the final result is quoted. Figure 1.2 shows a control volume representing a turbomachine, through which fluid passes at a steady rate of mass

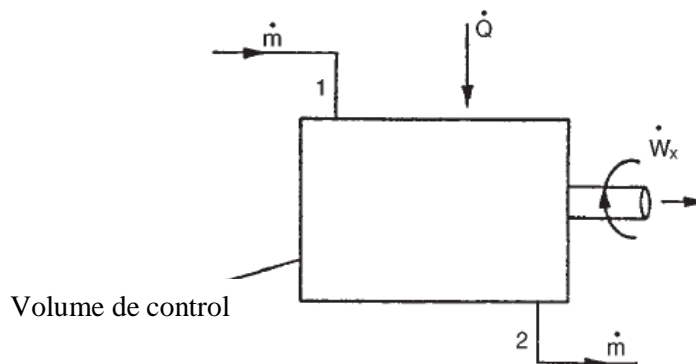


Figure 1.2. Volume de control montrant la convention des signes pour le transfert de travail et de chaleur.

flow \dot{m} , entering at position 1 and leaving at position 2. Energy is transferred from the fluid to the blades of the turbomachine, positive work being done (via the shaft) at the rate \dot{W}_x . In the general case positive heat transfer takes place at the rate \dot{Q} , from the surroundings to the control volume. Thus, with this sign convention the steady flow energy equation is

$$\dot{Q} - \dot{W}_x = \dot{m}[(h_2 - h_1) + \frac{1}{2}(c_2^2 - c_1^2) + g(z_2 - z_1)] \quad (1.5)$$

where h is the specific enthalpy, $(1/2) \cdot c^2$ the kinetic energy per unit mass and gz the potential energy per unit mass.

Apart from hydraulic machines, the contribution of the last term in eqn. (1.5) is small and usually ignored. Defining stagnation enthalpy by $h_0 = h + (1/2) \cdot c^2$ and assuming $g \cdot (z_2 - z_1)$ is negligible, eqn. (1.5) becomes

$$\dot{Q} - \dot{W}_x = \dot{m}(h_{02} - h_{01}) \quad (1.6)$$

Most turbomachinery flow processes are adiabatic (or very nearly so) and it is permissible to write $\dot{Q} = 0$. For work producing machines (turbines) $\dot{W}_x > 0$, so that

$$\dot{W}_x = \dot{W}_t = \dot{m}(h_{01} - h_{02}) \quad (1.7)$$

For work absorbing machines (compressors) $\dot{W}_x < 0$, so that it is more convenient to write

$$\dot{W}_c = -\dot{W}_x = \dot{m}(h_{02} - h_{01}) \quad (1.8)$$