

# On the inverse of a square matrix

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## Definitin and Examples

### Fact

Let  $A \in \mathcal{M}_n(\mathbb{R})$ . If  $\det(A) \neq 0$ , then  $A^{-1}$  exists. Moreover, the formula of  $A^{-1}$  is given by:

$$A^{-1} = \frac{1}{\det(A)} (\text{Com}(A))^t, \quad (1)$$

where  $\text{Com}(A)$  denotes the comatrice of  $A$ . If  $A^{-1}$  exists, we say that  $A$  is **invertible**. By French "inversible".

### Example

Let  $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathcal{M}_2(\mathbb{R})$ . We have

$$\det(A) = ad - cb \text{ and } A^{-1} = \frac{1}{ad - cb} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

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## Definitin and Examples

### Example

Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 8 & 8 & 9 \end{pmatrix} \in \mathcal{M}_3(\mathbb{R}).$$

By definition, we obtain

$$\begin{aligned} \det(A) &= \begin{vmatrix} + & - & + \\ 1 & 2 & 3 \\ 4 & 5 & 6 \\ 8 & 8 & 9 \end{vmatrix} = \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2 \begin{vmatrix} 4 & 6 \\ 8 & 9 \end{vmatrix} + 3 \begin{vmatrix} 4 & 5 \\ 8 & 8 \end{vmatrix} \\ &= -3 + 24 - 24 \\ &= -3 \neq 0. \end{aligned}$$

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From (1), we have

$$\begin{aligned} A^{-1} &= \frac{-1}{3} \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}^t \\ &= \frac{-1}{3} \begin{pmatrix} \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} & -\begin{vmatrix} 4 & 6 \\ 8 & 9 \end{vmatrix} & \begin{vmatrix} 4 & 5 \\ 8 & 8 \end{vmatrix} \\ -\begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 8 & 9 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 8 & 8 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} \end{pmatrix}^t \\ &= \frac{-1}{3} \begin{pmatrix} -3 & 12 & -8 \\ 6 & -15 & 8 \\ -3 & 6 & -3 \end{pmatrix}^t = \frac{-1}{3} \begin{pmatrix} -3 & 6 & -3 \\ 12 & -15 & 6 \\ -8 & 8 & -3 \end{pmatrix}. \end{aligned}$$

As required.

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## Problems

**Ex 01.** Consider the matrix

$$A = \begin{pmatrix} 1 & -\alpha & & & \\ & 1 & -\alpha & & \\ & & \ddots & \ddots & \\ & & & 1 & -\alpha \\ & & & & 1 \end{pmatrix}; \alpha \in \mathbb{R}$$

Prove that

$$A^{-1} = \begin{pmatrix} 1 & \alpha & \alpha^2 & \dots & \alpha^{n-1} \\ & 1 & \alpha & \dots & \alpha^{n-2} \\ & & \ddots & \ddots & \vdots \\ & & & 1 & \alpha \\ & & & & 1 \end{pmatrix}.$$

**Ex 02.** Let  $A, B \in \mathcal{M}_2(\mathbb{R})$ . Assume that one of the matrices  $A$  or  $B$  is invertible. Show that  $AB$  and  $BA$  have the same characteristic polynomial, i.e.,  $p_{AB}(x) = p_{BA}(x)$ .