English 1 for Master students

At Department of Mathematics
University 08 Mai 1945 Guelma
By Bellaouar D., bellaouar.djamel@univ-guelma.dz

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1 Basic arithmetic operations

- **Addition:** $3 + 5 = 8$ three plus five equals [= is equal to] eight.
- **Subtraction:** $3 - 5 = -2$ three minus five equals [= . . .] minus two.
- **Multiplication:** $3{\cdot}5 = 15$ three times five equals [= . . .] fifteen.
- **Division:** $\frac{3}{5} = 0.6$ three divided by five equals [= . . .] zero point six.
- $(2 - 3){\cdot}6 + 1 = -5$ two minus three in brackets times six plus one equals minus five.
- $\frac{1-3}{2+4} = -\frac{1}{3}$ one minus three over two plus four equals minus one third.
- $4!$ [$= 1{\cdot}2{\cdot}3{\cdot}4$] four factorial.

2 Exponentiation and Roots

1. 5^2 five squared
2. 5^3 five cubed
3. 5^4 five to the (power of) four
4. 5^{-1} five to the minus one
5. $\sqrt{3}$ the square root of three
6. $\sqrt[3]{64}$ the cube root of sixty four
7. $\sqrt[5]{32}$ the fifth root of thirty two
8. $(1 + 2)^{2+2}$ one plus two, all to the power of two plus two
9. $e^{\pi i} = -1$ e to the (power of) pi i equals minus one

In the complex domain the notation $\sqrt[n]{a}$ is ambiguous, since any non-zero complex number has n different n-th roots. For example, $\sqrt[4]{-4}$ has four possible values: $\pm 1 \pm i$ (with all possible combinations of signs).

3 Algebraic Expressions

$A = a^2$	capital a equals small a squared
$A = \sqrt{a}$	capital a equals the square root of small a
$a = x + y$	a equals x plus y
$b = x - y$	b equals x minus y
$c = x.y.z$	c equals x times y times z
$(z + y)\, z + xy$	x plus y in brackets times z plus x y
$x^2 + y^3 + z^5$	x squared plus y cubed plus z to the (power of) five
$x^n + y^n = z^n$	x to the n plus y to the n equals z to the n
$(x - y)^{3m}$	x minus y in brackets to the (power of) three m x minus y, all to the (power of) three m
$\binom{n}{m}$	(the binomial coefficient) n over m
$2^x 3^y$	two to the x times three to the y
$ax^2 + bx + c$	a x squared plus b x plus c
$\sqrt{x} + \sqrt[3]{y}$	the square root of x plus the cube root of y
$\sqrt[n]{x + y}$	the n-th root of x plus y
$\frac{a+b}{c-d}$	a plus b over c minus d

4 Indices

x_0	x zero
$x_1 + y_i$	x one plus y i
R_{ij}	(capital) R (subscript) i j; (capital) R lower i j
M_{ij}^k	(capital) M upper k lower i j; (capital) M superscript k subscript i j
$\sum_{i=0}^{n} a_i x^i$	sum of a i x to the i for i from zero to n; sum over i (ranging) from zero to n of a i (times) x to the i
$\prod_{m=1}^{+\infty} b_m$	product of b m for m from one to infinity; product over m (ranging) from one to infinity of b m
$q_i = \sum_{j=1}^{n} a_{ij} b_{jk}$	q i equals the sum of a i j times b j k for j from one to n; q i is equal to the sum over j (ranging) from one to n of a i j times b j k
$\sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$	sum of n over i x to the i y to the n minus i for i from zero to n

5 Fractions [= Rational Numbers]

- $\frac{1}{2}$ one half, $\frac{3}{8}$ three eighths
- $\frac{1}{3}$ one third, $\frac{26}{9}$ twenty-six ninths
- $\frac{1}{4}$ one quarter [=one fourth], $\frac{-5}{34}$ minus five thirty-fourths
- $\frac{1}{5}$ one fifth, $2\frac{3}{7}$ two and three sevenths
- $\frac{-1}{17}$ minus one seventeenth, $\frac{1}{5}$ one fifth

6 Complex Numbers

i	i
$3+4i$	three plus four i
$1-2i$	one minus two i
$\overline{1-2i}=1+2i$	the complex conjugate of one minus two i equals one plus two i

- The real part and the imaginary part of $3+4i$ are equal, respectively, to 3 and 4.

7 Inequalities

- $x > y$ x is greater than y
- $x \geq y$ x is greater (than) or equal to y
- $x < y$ x is smaller than y
- $x \leq y$ x is smaller (than) or equal to y
- $x > 0$ x is positive
- $x \geq 0$ x is positive or zero; x is non-negative
- $x < 0$ x is negative
- $x \leq 0$ x is negative or zero

8 Set theory

1. $x \in A$ x is an element of A; x lies in A; x belongs to A; x is in A
2. $x \notin A$ x is not an element of A; x does not lie in A; x does not belong to A; x is not in A
3. $x, y \in A$ (both) x and y are elements of A; . . . lie in A; . . . belong to A; . . . are in A
4. $x, y \notin A$ (neither) x nor y is an element of A; . . . lies in A; . . . belongs to A; . . . is in A
5. $\emptyset$ the empty set (= set with no elements)
6. $A = \emptyset$ A is an empty set
7. $A \neq \emptyset$ A is non-empty
8. $A \cup B$ the union of (the sets) A and B; A union B
9. $A \cap B$ the intersection of (the sets) A and B; A intersection B
10. $A \times B$ the product of (the sets) A and B; A times
11. $A \cap B = \emptyset$ A is disjoint from B; the intersection of A and B is empty
12. $\{x\ /\ ....\}$ the set of all x such that . . .
13. $\mathbb{N}$ the set of natural numbers, $\mathbb{Z}$ the set of integers
14. $\mathbb{C}$ the set of all complex numbers
15. $\mathbb{Q}$ the set of all rational numbers
16. $\mathbb{R}$ the set of all real numbers
17. $A \cup B$ contains those elements that belong to A or to B (or to both).
18. $A \cap B$ contains those elements that belong to both A and B
19. $A^n = \underbrace{A \times A \times ... \times A}_{n\text{-times}}$ contains all ordered n-tuples of elements of A.
20. $S \Rightarrow T$ S implies T; if S then T
21. $S \Leftrightarrow T$ S is equivalent to T; S iff T

- $\forall\ x \in A...$ for each [= for every] x in A . . .
- $\exists\ x \in A...$ there exists [= there is] an x in A (such that) . . .
- $\exists!\ x \in A...$ there exists [= there is] a unique x in A (such that) . . .
- $\nexists\ x \in A...$ there is no x in A (such that). . .

9 Limit

By definition, an infinite series of complex numbers $\sum_{n=1}^{\infty} a_n$ converges (to a complex number l) if the sequence of partial sums $s_n = a_1 + a_2 + ... + a_n$ has a finite limit (equal to l); otherwise it diverges.

- $\lim_{x \to 1} f(x) = 2$ the limit of f of x as x tends to one is equal to two.
- What is the sum $1 + 2 + 3 + \cdots$ equal to?

10 Divisibility

The multiples of a positive integer a are the numbers $a, 2a, 3a, 4a, ...$ If b is a multiple of a, we also say that a divides b, or that a is a divisor of b (notation: $a|b$). This is equivalent to $\frac{b}{a}$ being an integer.

Two integers a, b are congruent modulo a positive integer m if they have the same remainder when divided by m (equivalently, if their difference $a - b$ is a multiple of m).

- $a \equiv b \pmod{m}$ a is congruent to b modulo m
- $a \equiv b\,(m)$ a is congruent to b modulo m

11 Prime Numbers

An integer $n > 1$ is a prime (number) if it cannot be written as a product of two integers $a, b > 1$. If, on the contrary, $n = ab$ for integers $a, b > 1$, we say that n is a composite number. The list of primes begins as follows:

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61...$$

Note the presence of several "twin primes" (pairs of primes of the form p, p + 2) in this sequence:

$$11, 13 \qquad 17, 19 \qquad 29, 31 \qquad 41, 43 \qquad 59, 61, ...$$

Two fundamental properties of primes:

Theorem 1 *There are infinitely many primes.*

Theorem 2 *Every integer $n \geq 1$ can be written in a unique way as a product of distinct prime powers.*

12 Functions

- $f(x)$ f of x
- $g(x, y)$ g of x (comma) y
- $h(2x, 3y)$ h of two x (comma) three y
- $\sin x$ sine x
- $\cos x$ cosine x
- $\tan x$ tan x
- $\arcsin x$ arc sine x
- $\arccos x$ arc cosine x
- $\arctan x$ arc tan x
- $\sinh x$ hyperbolic sine x
- $\cosh x$ hyperbolic cosine x
- $\tanh x$ hyperbolic tan x
- $\sin x^2$ sine of x squared
- $\sin^2 x$ sine squared of x; sine x, all squared
- $\dfrac{x+1}{\tan(y^4)}$ x plus one, all over over tan of y to the four
- $3^{x-\cos(2x)}$ three to the (power of) x minus cosine of two x
- $e^{x^3+y^3}$ exponential of x cubed plus y cubed

13 Intervals

- (a, b) open interval a, b
- $[a, b]$ closed interval a, b
- $(a, b]$ half open interval a, b (open on the left, closed on the right)
- $[a, b)$ half open interval a, b (open on the right, closed on the left)

14 Derivatives

1. f' f prime; the first derivative of f
2. f'' f double prime; the second derivative of f
3. f''' the third derivative of f
4. $f^{(n)}$ the n-th derivative of f
5. $\frac{dy}{dx}$ d y by d x; the derivative of y by x
6. $\frac{d^2y}{dx^2}$ the second derivative of y by x; d squared y by d x squared
7. $\frac{\partial f}{\partial x}$ the partial derivative of f by x (with respect to x); partial d f by d x
8. $\frac{\partial^2 f}{\partial x^2}$ the second partial derivative of f by x (with respect to x); partial d squared f by d x squared
9. ∇f nabla f; the gradient of f
10. $\triangle f$ delta f

15 Integrals

$\int f(x)\,dx$	integral of f of x dx
$\int\limits_a^b t^2 dt$	integral from a to b of t squared dt
$\int\int_S h(x,y)\,dxdy$	double integral over S of h of x y dx dy

16 Greek letters used in mathematics

α	alpha	β	beta	γ	gamma	δ	delta
ϵ, ε	epsilon	ζ	zeta	η	eta	θ, ϑ	theta
ι	iota	κ	kappa	λ	lambda	μ	mu
ν	nu	ξ	xi	o	omicron	π, ϖ	pi
ρ, ϱ	rho	σ	sigma	τ	tau	υ	upsilon
ϕ, φ	phi	χ	chi	ψ	psi	ω	omega

B	Beta	Γ	Gamma	Δ	Delta	Θ	Theta
Λ	Lambda	Ξ	Xi	Π	Pi	Σ	Sigma
Υ	Upsilon	Φ	Phi	Ψ	Psi	Ω	Omega

17 Polynomial equations

A polynomial equation of degree $n \geq 1$ with complex coefficients

$$f(x) = a_0 x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0 = 0, \text{ where } a_0 \neq 0$$

has n complex solutions (= roots), provided that they are counted with multiplicities.

For example, a quadratic equation

$$f(x) = ax^2 + bx + c = 0 \ (a \neq 0)$$

can be solved by completing the square, i.e., by rewriting the left hand side as

$$a(x + \text{constant})^2 + \text{ another constant.}$$

This leads to an equivalent equation

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a},$$

whose solutions are

$$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a},$$

where $\Delta = b^2 - 4ac \ (= a^2 (x_1 - x_2)^2)$ is the discriminant of the original equation. More precisely,

$$ax^2 + bx + c = a(x - x_1)(x - x_2).$$

If all coefficients a, b, c are real, then the sign of Δ plays a crucial[1] rôle[2]:

- if $\Delta = 0$, then $x_1 = x_2$ $(= \frac{-b}{2a})$ is a double root;
- if $\Delta > 0$, then $x_1 \neq x_2$ are both real;
- if $\Delta < 0$, then $x_1 = \overline{x_2}$ are complex conjugates of each other (and non-real).

[1] essential, important

[2] role or rôle, both are true and have the same pronunciation.