University of Guelma **January 26th 2016**

Mathematics Department **First Exam of English 1 (Master 1)** **2 H**

Ex 01 (0.5×17 marks)**:** Complete the following notions by using the correspondent mathematical words and phrases.

1. Matrices with the property $A^*A = AA^*$ are said to be .....

2. Let $A \in M_n(\mathbb{R})$ be a square matrix. If $\det(A) \neq 0$, then A is .....

3. For square matrices A, the number $\rho(A) = \max_{\lambda \in \sigma(A)} |\lambda|$ is called the ..... of the matrix A.

4. Let $A \in M_n(\mathbb{R})$ be a square matrix. The ..... for A is to find solutions to the matrix equation $Ax = \lambda x$, where $\lambda \in \mathbb{K}$ and $x \in \mathbb{K}^n$ such that $x \neq 0$.

5. Recall that a function f is called an ..... function if $f(-x) = f(x)$. Similarly, f is called an ..... function if $f(-x) = -f(x)$.

6. A collection of vectors $\{v_1, v_2, ..., v_k\}$ is said to be an ..... set if $\langle v_i, v_j \rangle = 0$ for all $i \neq j$. If, in addition, $||v_i|| = 1$ for all $i = 1, 2, ..., k$, the set is called .....

7. A symmetric matrix $A \in M_n(\mathbb{R})$ is called ..... if

$$x^t A x > 0 \quad \text{for all } x \in \mathbb{R}^n.$$

8**.** A function $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is called a ..... function if for all $\theta \in [0, 1]$, and for any $x, y \in \mathbb{R}^n$ we have

$$f(\theta x + (1 - \theta) y) \leq \theta f(x) + (1 - \theta) f(y).$$

9. The inequality : $\|x + y\| \leq \|x\| + \|y\|$ which is known " ....."

10. A set A is said to be ..... iff A is contained in the range of some sequence (briefly, the elements of A can be put in a sequence).

11. In any metric space (E, d), every compact subset K is .....

12**.** (.....). If $f : [a; b] \longrightarrow \mathbb{R}$ is continuous and $f(a) < y < f(b)$, then y must be a value of f.

13**.** A topological space (X, τ) is T_2 (or Hausdorff) iff given $x \neq y$ in X, there exist ..... of x and y.

14. The closed graph theorem states the following: If X is a ..... space and Y is a ..... space, then the graph of a linear map T from X to Y is closed if and only if T is continuous.

Ex 02 (3.5 marks)**.** Write the following formulas in full form :

$$\sqrt[n]{n! + \overline{x}} \leq 1 \qquad i, e., \qquad \sum_{i=1}^{n} \frac{e^{x_i^2 + \cos\left(x_i^n\right)}}{\sqrt{\sin^3(x_i) - 1}} \qquad \frac{\pi - \varphi + \xi}{(\varepsilon + \rho + \phi)^{\omega}} \qquad f^{(n)} \notin A \qquad \frac{\partial f}{\partial x} \qquad A \cap B = \emptyset$$

Ex 03 (5.5 marks). Read carefully the following text.

The absolute value function on $\mathbb{R}$ and the modulus on $\mathbb{C}$ are denoted by $|.|$ and each gives a notion of lenght or distance in the corresponding space and permits the discussion of convergence of sequences in that space or continuity of functions on that space. In this work, we shall extend these concepts to a general linear space E. A seminorm on the linear space E is a function $p : E \to \mathbb{R}$ for which $p(\alpha x) = |\alpha| p(x)$ and $p(x + y) \leq p(x) + p(y)$ for all $\alpha \in \mathbb{K}$ and $x, y \in E$. The pair (E, p) is called a seminormed space. We study some properties concerning seminormed spaces, for example, a closed subspace of a seminormed space is complete but the reciprocal is false. Finally, we prove that a complete subspace of a normed space is closed.

1) Give a suitable title of the text.

2) Find a word or expression in the text which, in context, is similar in meaning to :

converse, full, series, notion, map (mapping), to expand.

3) Turn from active to passive the following statements:

a- We had studied some properties concerning seminormed spaces.

b- In this work, we shall extend these concepts to a general linear space E.

c- Hilbert let many problems without proof.

d- We may use the contraction mapping theorem to prove the existence and uniqueness of solutions.

4) Give the phonetic of the following words.

discussion, normed, general, called, value, properties.

Ex 04 (3 marks): Give one word (if it is possible in relation with mathematics) for every symbol.

ə	iː	ɪ	æ	e	ʌ	
ɔː	ɒ	aː	uː	ʊ	əː	
eɪ	eə	aɪ	ɔɪ	aʊ	ɪə	əʊ

Good luck.