University 08 Mai 1945 Guelma

Department of Mathematics **Master 1** **2019-2020**

English 1

By Dr. Bellaouar Djamel

Problem. Complete the following sentences by using the correspondant mathematical notions.

1) $\mathbb{N}$ The set of ……..……………………………

2) $\mathbb{Z}$ The set of ……………………………………

3) $\mathbb{R}$ The set of ………………. numbers

4) $\mathbb{C}$ The set of ……. ……..….. numbers

5) $|x|$ the……………….. …. of a real or complex numbers x.

6) $]a,b[$ an open … ………….……., $[a,b]$ a closed … ……………

7) $\overline{z}$ The ………..… … of a ……………………..… number z.

8) $\mathbb{R}^n$ *The* real euclidean space of ………………………………

9) $C([a,b])$ space of real or complex ……………………….....on the interval $[a,b]$.

10) $C^m([a,b])$ Space of m-times …………………………………………...

11) $L^2[a,b]$ Space of real- or complex ……………………………………………

12) $\{\alpha_1, \alpha_2, ..., \alpha_m\}$ **:** The set of m ………………………………………

A mapping with ………………… X and ………………….. in Y

$$f : X \longrightarrow Y$$

What the meaning of these symbols ?

$$\in \quad \subset \quad \cup \quad \cap$$

The ……………………... , the ……….…….., ……..……..… and … ………………

We denote by

$$A^t \quad A^* \quad A^{-1} \quad \det(A) \quad Cond(A) \quad \rho(A)$$

The ……………..., ……………., ……………., ………………., …………………... and the ……………………... of a matrix A.

<,> : ………………………..….. on a linear space or ……………………………….…

< x , y > : ……………………….……….. between x and y and for the space $C([a,b])$ the ……………………….. between f and g is given by

$$\langle f, g\rangle = \int_a^b f(x)\, g(x)\, dx$$

……………………….. coefficients:

$$\binom{m}{n} = \frac{m!}{n!\,(m-n)!}$$

The opposite of a derivative is the ……………………………………..
The ……………………………….. of a function $f(x)$ is denoted,

$$\int f(x)\,\mathrm{d}x.$$

…………………………. is …………………, that means:

$$\int [\alpha f(x) + \beta g(x)]\, dx = \alpha \int f(x)\, dx + \beta \int g(x)\, dx$$

…………………………... Consider a square matrix A. A nonzero vector x is an ………………. of the matrix with …………..… λ if $Ax = \lambda x$.
Let

$$p_A(\lambda) \quad = \quad (\lambda - 2)^3$$

We see that $\lambda = 2$ is an …………………….. of ………………..…. *3*.

…**Binomial**…. ……………….

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \ldots + nab^{n-1} + b^n$$

Integration by ……………..…, Integration by …………………

The ……………………… of x^2 in $(2+3x)^2$ is 9.

Factorizing a polynomial is the opposite of the ………………………..…

………………. **and** … ………………………………

$$(a+b)(c+d) = ac + ad + bc + bd$$
$$(a-b)(a+b) = a^2 - b^2$$
$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

A ………………………... equation is of the form :

$$y = ax^2 + bx + c$$

Where a, b and c are constants.

A …………………….. has the general form :

$$p_n(x) = a_0 + a_1 x + ... + a_n x^n$$

The polynomial has **………………………** n if its highest **…………….** is x^n

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The …………………..…..sign is defined as

$$\sum_{i=1}^{n} f(i) = f(1) + f(2) + ... + f(n-1) + f(n)$$

The **…………………..** notation is defined as follows

$n! = n.(n-1).(n-2)...3.2.1$, where n is an integer

The …………………..... of a function is denoted $f^{-1}(x)$ and has the property that

$$f^{-1}(f(x)) = f(f^{-1}(x)) = x.$$

The **…………..** or (…………..).. of a function $f(x)$ are the values of x when $f(x) = 0$.

A function is **………..** if $f(-x) = f(x)$ and ……………..….. if $f(-x) = - f(x)$

A …………..…. has the general form

$$y = mx + a,$$

where *a* and *m* are real numbers and *m* is the **…………..** of the **……………**

A fundamental **………………..** identity is

$$\cos^2 x + \sin^2 x = 1$$

………….. functions:

$$chx = \frac{e^x + e^{-x}}{2} \qquad shx = \frac{e^x - e^{-x}}{2}$$

…………..... rule

$$[f(g(x))]' = g'(x)\, f'(g(x))$$

The **………….....** of an equation or inequality in one unknown, say *x*, is the collection of all numbers that make the equation or inequality a true statement. Some times this set of numbers is called the **……………... set**.

Any ………………. set T of natural numbers contains a smallest element.

Theorem. The product of finitely many compact spaces is **………………**

Theorem

- Any nonempty subset of the …………….…… numbers is countable.
- Any subset of a countable set is …………………...

Theorem : The set of real numbers $\mathbb{R}$ is not …………………….

Theorem : An **………….** set in R is the union of a countable family of disjoint **……………** intervals.

Every **………………** of a convergent sequence converges to the same limit.

- A product is **……………** if and only if both factors are of like **…………..**

- $x < y : x$ is ………………. than *y*, $x > y : x$ is ……………… than *y*.
- $x \leq y : x$ is ……………….. *y*, $x \geq y : x$ is ………………….. *y*.

The **……………....** with **center** (1,1) and **…………….** 3 has the equation :

$$(x-1)^2 + (y-1)^2 \quad = \quad 9$$

Lemma : A convergent …………………... .. is bounded.

Definitions

The …………………...function, denoted by *log*, is defined by the following ……………... :

$$\log x = \int_1^x \frac{1}{t}\,dt, \quad x > 0$$

The ………………... of the logarithm function is called the …………………………… and is denoted by *exp*.

Five laws

1. ……………….. The limit of a sum is the ……………... of the limits.
2. ……………… The limit of a difference is the ……………… of the limits.
3. ……………….. The limit of a constant times a function is the ………………. the limit of the function.
4. ……………….. The limit of a product is the ………………. of the limits.
5. ……………….. The limit of a quotient is the ……………... of the limits (provided that the limit of the denominator is not 0)

Direct ……………………. property If f is a polynomial or a rational function and a is in the domain of f, then

$$\lim_{x \longrightarrow a} f(x) = f(a)$$

Definition A function f is ………………. from the ……………... at a number a if

$$\lim_{x \longrightarrow a^+} f(x) = f(a)$$

and is ……………. from the ………………….. at a if :

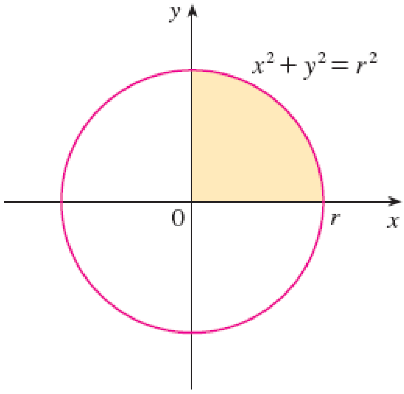
$$\lim_{x \longrightarrow a^-} f(x) = f(a)$$

Definition A function f is ……………….. on an interval if it is ………………at every number in the interval.

The Product Rule If f and g are both …………………., then

$$(f.g)' = f'g + g'f$$

Prove that the area of a ………….. with ………………. r is $S = \pi r^2$



Find the ………………. of convergence and the …………. of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$$

Find a ………………….. series representation for the function and determine the interval of convergence.

$$f(x) = \frac{x}{1-x}$$

Use differentiation to find a ……………… series representation for

$$f(x) = \frac{1}{(1+x)^2}$$

What is the ………………… of convergence?

Definition A ………………. E is called ……………. if and only if every Cauchy sequence in E converges to an element x of the space E.

Definition . In an ordered …………….. we define the ……………..… $|a|$ of a as :

$$|a| = \begin{cases} a \; ; a > 0 \\ -a \; ; \; a < 0 \\ 0 \; ; \; a = 0 \end{cases}$$

Each real number x has a corresponding ……………………, defined as

$$|x| = \max\{x, -x\} .$$

- $|x + y| \leq |x| + |y|$ is called the ………………………

The …………………of z is

$$|z| = \sqrt{x^2 + y^2}$$

and then we have the …………………. $d(a, b) = |a - b|$ between a and b.

$$\lim_{n \longrightarrow \infty} a_n = a$$

and also we say :

$$a_n \longrightarrow a \quad (a_n \text{ tends to } a)$$

as n …………... to ∞

Prove that a convergent ……………………. of real numbers is a Cauchy sequence.

Prove that if $p \geq 2$, then the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

is …………………….....

Discuss the …………………… of the series

$$\sum_{n=1}^{\infty} \frac{n^n}{n!}$$

Theorem (Least Upper Bound Principle). *A ………….… set of reals which is bounded above has the ……………………...*

The ……………. satisfies

$$||x|| \geq 0, \qquad ||x|| = 0 \Leftrightarrow x = 0 \in \mathbb{R}^n$$
$$||\lambda x|| = |\lambda| \; ||x||$$
$$||x + y|| \leq ||x|| + ||y||$$

and the …………………. satisfies

$$d(x, y) \geq 0, \qquad d(x, y) = 0 \Leftrightarrow x = y$$
$$d(x, y) = d(y, x)$$
$$d(x, z) \leq d(x, y) + d(y, z).$$

$$x^{(n)} \longrightarrow x \text{ in } \mathbb{R}^p \text{ iff } \left\| x^{(n)} - x \right\| \longrightarrow 0 \text{ in } \mathbb{R}$$

The half open ……………..]a,b] is neither ……………. nor ……………..
f is ………………..…. at a iff $f(x) \to f(a)$ as $x \to a$ in E.

A function f : [a,b] → R is ……………………. if and only if :

$$x \leq y \Rightarrow f(x) \leq f(y) \;\; ; x, y \in [a,b]$$

……………….. if and only if :

$$x \leq y \Rightarrow f(x) \geq f(y) \;\; ; x, y \in [a,b]$$

…………………….. if and only if it is either ……………. or ……………

Definition. *A map* $T : (X, d) \to (X, d)$ *on* a metric *space* (X, d) *is a* ……………………. *if and only if for some k,* $0 \leq k < 1$

$$\forall x, y \in X : d(Tx, Ty) \leq kd(x, y)$$

Theorem (………………….. Mapping Theorem). Suppose that $T : (X, d) \to (X, d)$ is a …………………….. on a (non-empty) complete metric space (X, d). Then T has a ……………………….. fixed point.

Let A and B be nonempty subsets of a ………………. space X, and define

$$A + B = \{x + y; x \in A \text{ and } y \in B\}$$

Prove that

(*i*) if A is open, then $A + B$ is …………………..,

(*ii*) if A is compact and B is closed, then $A + B$ is …………………....

The real number θ satisfying the equations

$$x = r\cos\theta \text{ and } y = r\sin\theta$$

is called an ……………….. of z, and denoted by $arg\ z$. Hence we can write z in …………….. form :

$$z = r(\cos\theta + i\sin\theta).$$

…………………………formula:

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta \text{ for every } n \in \mathbb{N} \text{ and } \theta \in \mathbb{R}$$

Find all the roots of the ……………… :

$$z^n = a + ib; a, b \in \mathbb{R}$$

Suppose that $z = x + iy$, where x, y are ……..…... Then the ……………….. function $exp(z)$ is defined for every ………………. number z by

$$e^z = e^x(\cos y + i\sin y)$$

The ………………… fraction expansion **:**

$$\sqrt{2} = 1 + \frac{1}{2+\frac{1}{2+\frac{1}{2+\dots}}}$$

Show that the **……………………... series**

$$\sum_{n=1}^{\infty}\frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

is ………………………….

Let *R* be an equivalence relation on a set *E*. That means
R (**1**) is ……………, (**2**) is ………………, (**3**) is ……………….

A **to the ………………….., A to the** ………

$$\mathbf{A}^n = \underbrace{\mathbf{A}\mathbf{A}\cdots\mathbf{A}}_{n \text{ times}}.$$

Matrix Inversion

• A matrix *A* is said to be **…………………..** if there exists *B* such that

$$A.B = B.A = I_n$$

B is denoted A^{-1} and is unique.
• If *det*(*A*) = 0, then the matrix is not **…………………(is ………………..)**

The ………………….. of *n* matrix *A* can be found by considering the transpose of the cofactors matrix divided by the determinant of *A*.

$$A^{-1} = \frac{1}{\det(A)}C^t$$

A vector *v* is a ………………… of the vectors *x, y* and *z* if it can be written as :

$$v = \alpha x + \beta y + \gamma z \;;$$

where α,β,γ are constants

A set of vectors *u,v, w* are linearly ………………… if the only constants α,β,γ that satisfy

$$\alpha u + \beta v + \gamma w = 0$$

are α = β = γ = 0.

Determine whether or not the set *B* is linearly ……………………..

• When *A* is a square matrix, each of the following statements is equivalent to saying that *A* is ………………………

- The columns of *A* form a linearly independent set.
- The rows of *A* form a linearly independent set.

The set

$$\{1, x, x^2, \ldots, x^n\}$$

is a ………………… for the vector space of ……………..….. having degree *n* or less.

The ……………………….. of a vector space *V* is defined by :

dim V = number of vectors in any basis for *V* .

The .... defined by :

$$\mathbf{x}^T\mathbf{y} = \sum_{i=1}^{n} x_i y_i \in \Re \quad \text{and} \quad \mathbf{x}^*\mathbf{y} = \sum_{i=1}^{n} \bar{x}_i y_i \in \mathcal{C}$$

Determine the ……… (………………..) polynomial for the matrix

$$\mathbf{A} = \begin{pmatrix} 5 & 1 & 2 \\ -4 & 0 & -2 \\ -4 & -1 & -1 \end{pmatrix}$$

• The base of the …………………………., named in honor of Euler. It appears in many mathematical contexts involving ……………… and ……………….., and can be defined by

$$e = \lim_{n \longrightarrow +\infty} \left(1 + \frac{1}{n}\right)^n$$

The mean value Theorem

For any polynomial function *p*(*x*) with real coefficients, suppose that for $a \neq b$, $p(a)$ and $p(b)$ are of opposite ……………….. Then the function has a real …………… between *a* and *b*.

The Fundamental Theorem of Algebra

Every ………………………….. of degree *n*, with $n \geq 1$, has at least one zero in the system of …………………….. numbers.

…………………. Set

A set of a ………………….. H satisfying :

$$\langle e_i, e_j \rangle = \begin{cases} 1\ ; i = j \\ 0\ ;\ i \neq j \end{cases}$$

- ……………………….. equation :

A *differential equation* involving a function of more than one variable, and hence partial derivatives.

- ……………………… : A linear combination of nonnegative, integer powers of a variable (or unknown) *x*,

$$p(x) = a_0 + a_1 x + \ldots + a_n x^n$$

Given two sets, *A* and *B*, their ……………….., $D = A - B$, is the set of all elements of *A* not found in *B*.

- ………………………………..

Given two sets, A and B, their ……………………… is defined as

$$A \setminus B = (A - B) \cup (B - A)$$

- ………………….. function : The ***characteristic function*** of a set *A* is defined by

$$x_A(x) = \begin{cases} 1\ ; x \in A \\ 0\ ; x \notin A \end{cases}$$

Definition : For a *linear* operator $A : X \to X$ a scalar λ is called …………………. of *A* if there is a nontrivial solution *x* to the equation $Ax = \lambda x$. Such an *x* is called ……………………… .corresponding to λ. The subspace of all solutions of the equation $(A - \lambda I)\, x = 0$ is called the ………………….. of *A* corresponding to λ.

Matrix Decomposition Theorem

Let *P* be a matrix of ……………………… of a given matrix *A* and *D* a matrix of the corresponding …………………. Then *A* can be written as

$$A = PDP^{-1}$$

where *D* is a ……………………… matrix.

Absolute value of an operator

Let A be a bounded linear operator on a Hilbert space, H. Then the ……………………value of A is given by :

$$|A| = \sqrt{A^* A}$$

where A^* is the ……………….. of A.

The difference between the …………….. value of a number x and an …………… value a is called the absolute error of the …………………… value, *i.e,*

$$\Delta_a = |x - a|$$

The quotient

$$\delta_a = \frac{\Delta_a}{|a|}$$

is called the …………… error.

- ………………… series : A *series* that alternates signs, *i.e.*, of the form

$$\sum (-1)^n a_n \;; a_n \geq 0$$

Banach space : A ……………... *vector space* which is *complete* in the *metric* defined by its norm.

A subset E of a *vector space* V is called a ……………... of V if each *vector* $x \in V$ can be uniquely written in the form :

$$x = \sum_{i=1}^{n} \alpha_i e_i \;; e_i \in E$$

Bounded linear …………….. : A *bounded linear* ... …………… from a *normed* linear space E to another *normed* linear space F is a map $T : E \rightarrow F$ which satisfies

(i) $T(\alpha x + \beta y) = \alpha T(x) + \beta T(y)$; for all $x, y \in E, \alpha, \beta \in \mathrm{R}$; ……………

$$(ii) \quad \|T(x)\|_F \leq c\,\|x\|_E$$

for some constant $c \geq 0$, for all $x \in \mathrm{E}$; ………………………

Boundedness is equivalent to ……………………...

Characteristic equation: For …………………….. A the equation $det(A - \lambda.I) = 0$ is called the characteristic equation.

Complete space

A …………… *space* (M, d) is *complete* if every ……………... ………...{ X_n } in M converges in M. That is, there is $x \in M$ such that $d(X_n, x) \to 0$, as $n \to \infty$.

The …………………… : $f * g$ of two functions f and g is given by :

$$(f * g)(x) \quad = \quad \int f(x-y)\, g(y)\, dy$$

Let { $e_1, e_2, \ldots$} be an *orthonormal basis* in a *Hilbert space* (H,<,>). Then every $x \in H$ can be written as a ………………... series

$$x \quad = \quad \sum_{i \in I} \langle x, e_i \rangle\, e_i$$

The coefficients < x, e_i > are called the *Fourier* ……………. of x.

The ……………. norm satisfies

$$\|Ax\| \leq \|A\| \,.\, \|x\| \,;\, \forall\ x \in \mathbb{R}^n$$

Theorem : Every complex ……………………… is similar to an upper triangular matrix

Algebraic number : A complex number which is a zero of a …………………….. with rational coefficients

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An …………………..bound on a set S of real numbers is a number u so that $u \geq s$ for all $s \in S$. If such a u exists, S is said to be bounded **………………** by u.

……………… set : A set denoted ∅ which has no elements.

……………. : Two sets A and B which have the **same** elements; that is, if for all x, $x \in A$ …………………….. $x \in B$.

……………..sets : Two sets A and B such that there exists a bijection $f : A \to B$.

The differential equation :

$$ay'' + by' + cy = f(x)$$

has the solution

$$y(x) = y_h(x) + y_p(x)$$

where $y_h(x)$ is the ……………… solution to the homogeneous equation and $y_p(x)$ is a ……………. solution to the complete equation.

===**end**

Solution

1) $\mathbb{N}$ The set of ……… natural numbers
2) $\mathbb{Z}$ The set of ………… integers.
3) $\mathbb{R}$ The set of … real.. numbers
4) $\mathbb{C}$ The set of … complex ….. numbers
5) $|x|$ … the absolute value …. of a real or complex numbers x.
6) $]a,b[$ an open … interval., $[a,b]$ a closed … interval …..

7) $\overline{z}$ The … Conjugate … of a … complex … number z.

8) $\mathbb{R}^n$ *The* real euclidean space of … *n*-dimensional…..

9) $C([a,b])$ space of real or complex … continuous functions...on the interval $[a,b]$.

10) $C^m([a,b])$ Space of *m*-times … continuously differentiable functions.
11) $L^2[a,b]$ Space of real- or complex … square-integrable functions

12) $\{\alpha_1, \alpha_2, ..., \alpha_m\}$: The set of *m* …… elements………

A mapping with domain *X* and range in *Y*

$$f : X \longrightarrow Y$$

What the meaning of these symbols ?

$$\in \quad \subset \quad \cup \quad \cap$$

The …element inclusion... and the …set inclusion…, … union… and … intersection …

We denote by

$$A^t \quad A^* \quad A^{-1} \quad \det(A) \quad Cond(A) \quad \rho(A)$$

The transpose, adjoint, inverse, determinant, condition number and the spectral radius of a matrix *A*.

$\langle,\rangle$: ….scalar product….. on a linear space or …..inner product…

< x , y > : is the inner product between x and y and for the space $C([a,b])$ the inner product between f and g is given by

$$\langle f, g\rangle = \int_a^b f(x)\,g(x)\,dx$$

…Binomial….. coefficients :

$$\binom{m}{n} = \frac{m!}{n!\,(m-n)!}$$

The opposite of a derivative is the indefinite integral…..
The indefinite integral of a function $f(x)$ is denoted,

$$\int f(x)\,\mathrm{d}x.$$

integration…. is linear, that means :

$$\int [\alpha f(x) + \beta g(x)]\,dx = \alpha \int f(x)\,dx + \beta \int g(x)\,dx$$

Eigenvalues and Eigenvectors. Consider a square matrix A. A nonzero **vector**… x is an …**Eigenvector**…. of the matrix with …**Eigenvalue**… λ if

$$Ax = \lambda x$$

$$p_A(\lambda) \quad = \quad (\lambda - 2)^3$$

We see that $\lambda = 2$ is an ……**Eigenvalue**….. of …**multiplicity**…. *3*.

…Binomial…. Expansion

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \ldots + nab^{n-1} + b^n$$

Integration by **…substitution…**, Integration by **…parts**

The **coefficient** of x^2 in $(2+3x)^2$ is 9.

Factorizing a polynomial is the opposite of the …**expansion**…

expansion. and … Factorization

$$(a+b)(c+d) = ac + ad + bc + bd$$
$$(a-b)(a+b) = a^2 - b^2$$
$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

A ..**quadratic**.. equation is of the form :

$$y = ax^2 + bx + c$$

Where a, b and c are constants.

A …**polynomial**….. has the general form :

$$p_n(x) = a_0 + a_1 x + \ldots + a_n x^n$$

The polynomial has **…degree…** n if its highest **power** is x^n

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The …**summation**…..sign is defined as

$$\sum_{i=1}^{n} f(i) = f(1) + f(2) + \ldots + f(n-1) + f(n)$$

The **factorial**. notation is defined as follows

$$n! = n.(n-1).(n-2)...3.2.1 \qquad \text{where } n \text{ is an integer}$$

The …**inverse**.. of a function is denoted $f^{-1}(x)$ and has the property that

$$f^{-1}(f(x)) = f(f^{-1}(x)) = x$$

The **roots** or (zeros).. of a function $f(x)$ are the values of x when $f(x) = 0$.

A function is **even** if $f(-x) = f(x)$ and …**odd**….. if $f(-x) = -f(x)$

A …**line**…. has the general form

$$y = mx + a$$

Where a and m are real numbers and m is the **slope** of the …**line**.

A fundamental **trigonometric** identity is

$$\cos^2 x + \sin^2 x = 1$$

hyperbolic functions

$$chx = \frac{e^x + e^{-x}}{2} \qquad shx = \frac{e^x - e^{-x}}{2}$$

chain.. rule

$$[f(g(x))]' = g'(x) f'(g(x))$$

The **solution.** of an equation or inequality in one unknown, say *x*, is the collection of all numbers that make the equation or inequality a true statement. Some times this set of numbers is called the **…solution. set**.

Any nonempty set T of natural numbers contains a smallest element.

Theorem The product of finitely many compact spaces is **compact**

Theorem

- Any nonempty subset of the …**natural**… numbers is countable.
- Any subset of a countable set is …**countable.**

Theorem : The set of real numbers R *is not countable.*

Theorem : An **open** set in R is the union of a countable family of disjoint **open** intervals.

Every **subsequence** of a convergent sequence converges to the same limit.

- A product is **positive** if and only if both factors are of like **sign…**

- $x < y$: *x* is strictly less than *y*, $x > y$: *x* is strictly larger than *y*.
- $x \leq y$: *x* is less or equals *y*, $x \geq y$: *x* is larger or equals *y*.

The **circle**.. with **center** (1,1) and **radius** 3 has the equation :

$$(x-1)^2 + (y-1)^2 = 9$$

Lemma : A convergent …**sequence** .. is bounded.

Definitions

The **natural logarithm function**, denoted by *log*, is defined by the following **formula**… :

$$\log x = \int_1^x \frac{1}{t}\,dt, \; x > 0$$

- The inverse.. of the logarithm function is called the **exponential function** and is denoted by *exp*.

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five laws

1. Sum Law The limit of a sum is the sum. of the limits.
2. Difference Law The limit of a difference is the difference. of the limits.
3. Constant Multiple Law The limit of a constant times a function is the **constant times**.. the limit of the function.
4. Product Law The limit of a product is the …product of the limits.
5. Quotient Law The limit of a quotient is the quotient. of the limits (provided that the limit of the denominator is not 0)

Direct …substitution…. property If f is a polynomial or a rational function and *a* is in the domain of f, then

$$\lim_{x \longrightarrow a} f(x) = f(a)$$

Definition A function f is continuous. from the **right**.. at a number *a* if

$$\lim_{x \longrightarrow a^+} f(x) = f(a)$$

and is continuous from the …left… at *a* if :

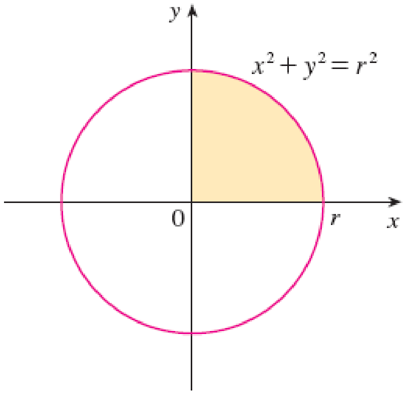
$$\lim_{x \longrightarrow a^-} f(x) = f(a)$$

Definition A function f is **continuous..** on an interval if it is **continuous** at every number in the interval.

The Product Rule If f and g are both differentiable, then

$$(f.g)' = f'g + g'f$$

Prove that the area of a circle . with radius *r* is $S = \pi r^2$



Find the radius of convergence and the domain of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$$

Find a …power . series representation for the function and determine the interval of convergence.

$$f(x) = \frac{x}{1-x}$$

Use differentiation to find a …power series representation for

$$f(x) = \frac{1}{(1+x)^2}$$

What is the radius of convergence?

Definition A **normed space** *E* is called **complete** if and only if every Cauchy sequence in *E* converges to an element *x* of the space *E*.

Definition . In an ordered …**field** we define the …**absolute value**… $|a|$ of *a* as :

$$|a| = \begin{cases} a \; ; a > 0 \\ -a \; ; \; a < 0 \\ 0 \; ; \; a = 0 \end{cases}$$

Each real number *x* has a corresponding ...*absolute value*., defined as

$$|x| = \max\{x, -x\} \; .$$

- $|x + y| \leq |x| + |y|$ is called the ...*triangle inequality*..

The *modulus* of *z* is

$$|z| = \sqrt{x^2 + y^2}$$

And then we have the …distance…. $d(a, b) = |a - b|$ between a and b.

$$\lim_{n \longrightarrow \infty} a_n = a$$

And also we say :

$$a_n \longrightarrow a \quad (a_n \text{ tends to } a)$$

as n tends.. to ∞

Prove that a convergent …sequence…. of real numbers is a Cauchy sequence

Prove that if $p \geq 2$, then the series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

is convergent..

Discuss the convergence of the series

$$\sum_{n=1}^{\infty} \frac{n^n}{n!}$$

Theorem (Least Upper Bound Principle). *A* ***nonempty****… set of reals which is bounded above has the* ***least upper bound.***

The **norm** satisfies

$$||x|| \geq 0, \qquad ||x|| = 0 \Leftrightarrow x = 0 \in \mathbb{R}^n$$
$$||\lambda x|| = |\lambda| \; ||x||$$
$$||x + y|| \leq ||x|| + ||y||$$

And the …**distance**… satisfies

$$d(x, y) \geq 0, \qquad d(x, y) = 0 \Leftrightarrow x = y$$
$$d(x, y) = d(y, x)$$
$$d(x, z) \leq d(x, y) + d(y, z).$$

$$x^{(n)} \longrightarrow x \text{ in } \mathbb{R}^p \text{ iff } \left\| x^{(n)} - x \right\| \longrightarrow 0 \text{ in } \mathbb{R}$$

The half open interval $]a,b]$ is neither **open**. nor **closed**…

f is …**continuous**…. at a iff $f(x) \rightarrow f(a)$ as $x \rightarrow a$ in E.

A function f : [a,b] → R is **increasing.** if and only if :

$$x \le y \Rightarrow f(x) \le f(y) \ \ ; x, y \in [a, b]$$

decreasing.. if and only if :

$$x \le y \Rightarrow f(x) \ge f(y) \ \ ; x, y \in [a, b]$$

monotonic.. if and only if it is either …increasing. or decreasing…

Definition. *A map* $T : (X, d) \to (X, d)$ *on a metric space* (X, d) *is a contraction if and only if for some k,* $0 \le k < 1$

$$\forall x, y \in X : d(Tx, Ty) \le k d(x, y)$$

Theorem (Contraction. Mapping Theorem). Suppose that $T : (X, d) \to (X, d)$ is a **Contraction** on a (non-empty) complete metric space (X, d). Then T has a **unique**. fixed point.

Let A and B be nonempty subsets of a normed. space X, and define

$$A + B = \{x + y; x \in A \text{ and } y \in B\}$$

Prove that

(*i*) if A is open, then $A + B$ is …open..;

(*ii*) if A is compact and B is closed, then $A + B$ is closed..

The real number θ satisfying the equations

$$x = r \cos \theta \text{ and } y = r \sin \theta$$

is called an argument of z, and denoted by *arg z*. Hence we can write z in polar form :

$$z = r(\cos \theta + i \sin \theta).$$

De Moivre's formula.,

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta \text{ for every } n \in \mathbb{N} \text{ and } \theta \in \mathbb{R}$$

Find all the roots of the equation :

$$z^n = a + ib; a, b \in \mathbb{R}$$

Suppose that $z = x + iy$, where x, y are reals . Then the **exponential** function $exp(z)$ is defined for every complex number z by

$$e^z = e^x (\cos y + i \sin y)$$

The **continued fraction** expansion :

$$\sqrt{2} = 1 + \frac{1}{2+\frac{1}{2+\frac{1}{2+\dots}}}$$

Show that the **harmonic.. series**

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

is …divergent.

Let *R* be an equivalence relation on a set *E*. That means *R* (**1**) is reflexive, (**2**) is symmetric, (**3**) is transitive.

A **to the power of** *n*, A **to the** *n*

$$\mathbf{A}^n = \underbrace{\mathbf{A}\mathbf{A}\cdots\mathbf{A}}_{n \text{ times}}.$$

Matrix Inversion

• A matrix *A* is said to be **invertible** if there exists *B* such that

$$A.B = B.A = I_n$$

B is denoted A^{-1} and is unique.

• If *det*(*A*) = 0, then the matrix is not **invertible.**

The inverse. of *n* matrix *A* can be found by considering the transpose of the cofactors matrix divided by the determinant of *A*.

$$A^{-1} = \frac{1}{\det(A)} C^t$$

A vector *v* is a **combination** of the vectors *x*, *y* and *z* if it can be written as :

$$v = \alpha x + \beta y + \gamma z \ ;$$

where α,β,γ are constants

A set of vectors *u*,*v*, *w* are linearly independent if the only constants α,β,γ that satisfy

$$\alpha u + \beta v + \gamma w = 0$$

Are α = β = γ = 0.

Determine whether or not the set *B* is linearly independent.

• When A is a square matrix, each of the following statements is equivalent to saying that A is invertible

- The columns of A form a linearly independent set.
- The rows of A form a linearly independent set.

The set

$$\{1, x, x^2, \ldots, x^n\}$$

is a …basis. for the vector space of **polynomial….** having degree n or less.

The **dimension** of a vector space V is defined by :

$$\dim V = \text{number of vectors in any basis for } V.$$

The ...inner product.. defined by :

$$\mathbf{x}^T\mathbf{y} = \sum_{i=1}^{n} x_i y_i \in \Re \quad \text{and} \quad \mathbf{x}^*\mathbf{y} = \sum_{i=1}^{n} \bar{x}_i y_i \in \mathcal{C}$$

Determine the **minimum (characteristic**). polynomial for the matrix

$$\mathbf{A} = \begin{pmatrix} 5 & 1 & 2 \\ -4 & 0 & -2 \\ -4 & -1 & -1 \end{pmatrix}$$

• The base of the NATURAL LOGARITHM, named in honor of Euler. it appears in many mathematical contexts involving LIMITS and DERIVATIVES, and can be defined by

$$e = \lim_{n \longrightarrow +\infty} \left(1 + \frac{1}{n}\right)^n$$

The mean value Theorem

For any polynomial function $p(x)$ with real coefficients, suppose that for $a \neq b$, $p(a)$ and $p(b)$ are of opposite **signs**. Then the function has a real **zero** between a and b.

The Fundamental Theorem of Algebra

Every **polynomial**. of degree n, with $n \geq 1$, has at least one zero in the system of **complex** numbers.

orthonormal. Set

A set of a ***Hilbert space*** H satisfying :

$$\langle e_i, e_j \rangle = \begin{cases} 1\ ; i = j \\ 0\ ;\ i \neq j \end{cases}$$

- partial differential equation :

A *differential equation* involving a function of more than one variable, and hence partial derivatives.

- polynomial : A linear combination of nonnegative, integer powers of a variable (or unknown) *x*,

$$p(x) = a_0 + a_1 x + ... + a_n x^n$$

Given two sets, *A* and *B*, their ***difference***, $D = A - B$, is the set of all elements of *A* not found in *B*.

- *symmetric difference*

Given two sets, A and B, their ***symmetric difference*** is defined as

$$A \setminus B = (A - B) \cup (B - A)$$

- *characteristic* function : The ***characteristic function*** of a set *A* is defined by

$$x_A(x) = \begin{cases} 1\ ; x \in A \\ 0\ ; x \notin A \end{cases}$$

Definition : For a *linear* operator $A : X \to X$ a scalar λ is called ***eigenvalue.*** of *A* if there is a nontrivial solution *x* to the equation $Ax = \lambda\, x$. Such an *x* is called ***eigenvector*** .corresponding to λ. The subspace of all solutions of the equation $(A - \lambda I)\, x = 0$ is called the ***eigenspace*** of *A* corresponding to λ.

Matrix Decomposition Theorem

Let *P* be a matrix of **eigenvectors**. of a given matrix *A* and *D* a matrix of the corresponding **eigenvalues**. Then *A* can be written as

$$A = PDP^{-1}$$

Where *D* is a **diagonal** matrix.

Absolute value of an operator

Let *A* be a *bounded linear operator* on a Hilbert space, *H*. Then the *absolute value* of *A* is given by :

$$|A| = \sqrt{A^* A}$$

where A^* is the **adjoint** of *A*.

The difference between the **exact**. value of a number *x* and an **approach** value *a* is called the *absolute error* of the **approach** value, *i.e*

$$\Delta_a = |x - a|$$

The quotient

$$\delta_a = \frac{\Delta_a}{|a|}$$

Is called the **relative error**.

- alternating series : A *series* that alternates signs, *i.e*., of the form

$$\sum (-1)^n a_n \; ; a_n \geq 0$$

Banach space : A ***normed***. *vector space* which is *complete* in the *metric* defined by its norm.

A subset *E* of a *vector space V* is called a **basis**. of *V* if each *vector* $x \in V$ can be uniquely written in the form :

$$x = \sum_{i=1}^{n} \alpha_i e_i \; ; e_i \in E$$

Bounded linear operator : A *bounded linear* ... **operator** from a *normed* linear space *E* to another *normed* linear space *F* is a map $T : E \to F$ which satisfies

(ii) $T(\alpha x + \beta y) = \alpha T(x) + \beta T(y)$; for all $x, y \in E, \alpha, \beta \in$ R ; **linearity**

$$(ii) \quad \|T(x)\|_F \quad \leq \quad c\|x\|_E$$

for some constant $c \geq 0$, for all $x \in$ E ; **boundedness.**

Boundedness is equivalent to **continuity.**

Characteristic equation : For **an operator**. *A* the equation $det(A - \lambda.I) = 0$.

Complete space

A ***metric*** *space* (M, d) is *complete* if every ***cauchy. sequence*** $\{ X_n \}$ in *M* converges in *M*. That is, there is $x \in M$ such that $d(X_n, x) \to 0$, as $n \to \infty$.

The ***convolution product*** *:* $f * g$ of two functions *f* and *g* is given by :

$$(f * g)(x) = \int f(x-y)\, g(y)\, dy$$

Let $\{ e_1, e_2, \ldots \}$ be an *orthonormal basis* in a *Hilbert space* $(H,<,>)$. Then every $x \in H$ can be written as a **fourier**. series

$$x = \sum_{i \in I} \langle x, e_i \rangle\, e_i$$

The coefficients $< x, e_i >$ are called the *fourier* ***coefficients*** of *x*.

The **matrix** norm satisfies

$$\|Ax\| \leq \|A\| \,.\, \|x\| \,;\, \forall\ x \in \mathbb{R}^n$$

Theorem : Every complex **square matrix** is similar to an upper triangular matrix

Algebraic number : A complex number which is a zero of a **polynomial**. with rational coefficients

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An **upper** *bound* on a set *S* of real numbers is a number *u* so that $u \geq s$ for all $s \in S$. If such a *u* exists, *S* is said to be bounded **above** by *u*.

Nonempty set : A set denoted Ø which has no elements.

Equal sets : Two sets *A* and *B* which have the **same** elements; that is, if for all *x*, $x \in A$ **if and only if** $x \in B$.

equivalent sets : Two sets *A* and *B* such that there exists a bijection $f : A \to B$.

The differential equation :

$$ay'' + by' + cy = f(x)$$

has the solution

$$y(x) = y_h(x) + y_p(x)$$

where $y_h(x)$ is the **general** solution to the homogeneous equation and $y_p(x)$ is a ………**particular** solution to the complete equation.