Exercise o1(0.25 \times 8 marks): Re-write the following sentences in ordinary English.

- [w1t∫ kʌm f3:st ðə 't∫1k1n ɔ:r ði:, eg]
- [tə'de1 a1 sə'dzest ə,sʌbst1'tju:∫ən v sʌm 'kwest∫ənz]

Exercise o2 (4 marks): Write in full form

$$|g.f| = g.f$$
 and $\left(\frac{|g|}{||g||_q}\right)^q = \left(\frac{|f|}{||f||_p}\right)^p a.e$

and

$$e^A = \lim_{n \longrightarrow \infty} \sum_{k=0}^n \frac{A^k}{k!}.$$

Exercise o3 (8 marks): i) Translate the following abstract in French language.

Abstract. Separable metric space possesses special properties that some of problem in analysis can be solved in these spaces easily. The question which appears basically is what properties are needed for metric spaces to be separable. We will answer to this question exactly in this article and even will get a special structure for separable metric spaces that are the same topological and algebraic structure of real numbers.

ii) Give the phonetic symbols of the previous abstract.

Exercise o4 (4 marks): Translate in English language.

a) Dans tous les exercices, on se place dans un espace métrique (E, d). Les parties de E seront notées A, B, etc., les points de E seront notés x, y, etc.

b) Suites.

Exprimer la convergence d'une suite de points de E à l'aide des boules ouvertes de E. En déduire que toute sous-suite d'une suite convergente est convergente.

c) Un voisinage d'un point a est une partie de E contenant une boule ouverte centrée en a.

d) **Définition.** Un espace métrique est un ensemble E muni d'une fonction $d: E \times E \longrightarrow \mathbb{R}_+$ vérifiant pour tout triplet $(x, y, z) \in E$, on a

$$\Diamond) \ d(x,y) = 0 \iff x = y \text{ (positivité)}$$

 $\Diamond) \ d(x, y) = d(y, x) \quad \text{(symétrie)}$

$$\Diamond) \ d(x,z) \leq d(x,y) + d(y,z) \quad \text{(inégalité triangulaire)}.$$

Une telle fonction est appelée distance sur E.

Good Luck

Mathematics Department, Master 1

Inner Product Space

In mathematics, a vector space or function space in which an operation for combining two vectors or functions (whose result is called an inner product) is defined and has certain properties. Such spaces, an essential tool of functional analysis and vector theory, allow analysis of classes of functions rather than individual functions. In mathematical analysis, an inner product space of particular importance is a Hilbert space, a generalization of ordinary space to an infinite number of dimensions.

A point in a Hilbert space can be represented as an infinite sequence of coordinates or as a vector with infinitely many components. The inner product of two such vectors is the sum of the products of corresponding coordinates. When such an inner product is zero, the vectors are said to be orthogonal. Hilbert spaces are an essential tool of mathematical physics.

Exercise o1 : (8 marks)

- 1) Give another title of the text. (1 mark).
- 2) Find a word or expression in the text which, in context, is similar in meaning to :

Farness, Series, unlimited, interior, boundless (2.5 marks)

3) Turn from passive into active the following sentence (2 marks)

- A point in a Hilbert space can be represented as an infinite sequence of coordinates.

- 4) Re-write the following words in ordinary English (2.5 marks)
 - [INI'kwplItI], [I'nI∫əl], [pplI'nəʊm1əl], [ʌn'baʊnd1d]
 - ['ældʒ1brə], [1'sen∫əl], ['negl1dʒəbl], [d1fəren∫1'e1∫ən], ['prɒpət1]

Exercise o2 (4 marks) : Translation

Translate the following paragraph in English language.

Théorème. Pour qu'un espace métrique (E,d) soit complet, il faut et il suffit que, toute suite décroissante de boules fermées de rayons tendant vers zéro admette une intersection non vide. (Plus précisément, cette intersection est réduite à un seul élément).

Exercise o3 (8 marks) : Vocabulary and Grammar

Choose the correct item.

1) Sarah is the prettiest girlour school. then in of
2) Paul's car isthan Tom's. fast fastest faster
3) This dress is the in the shop. more expensive most expensive expensive
4) Bob the car at the moment. washes is washing wash
5) The sun in the west. set is setting sets
6) Peter is asas Sally . clever cleverer cleverest
7) Ann has two brothersof them are older than him. none all both
8) I was hungry so I madea sandwish . me myself my
9) She works in a bank,? does she isn't she doesn't she
10) That ispen. Tom's Tom Toms'
11) How would you feel if youyour car? crash will crash crashed
12) Iread or write when I was four years old. can't couldn't wasn't able
13) Yoube rude to your parents. must musn't coudn't
14) Ibuy a new coat if I had enough money. would must will
15) He had studied hard so heanswer all the questions in the test.
was able to is able to can

16) She's known mea long time. ago for since

Page 2-2