Sheet1 : Infinite Series

EXERCICE 1.

A**:** Write

— $\sum_{n=1}^{\infty} ar^{n-1}$ as a series that starts at $n=0$, $\sum_{n=1}^{\infty} \frac{n^2}{1-3^{n+1}}$ as a series that starts at $n=3$.

B**:** Determine if the following series converges or diverges. If it converges determine its sum.

— (1) $\sum_{n=2}^{\infty} \dfrac{1}{n^2-1}$, (2) $\sum_{n=0}^{\infty}(-1)^n$ $(3)^{(*)}$ $\sum_{n=0}^{\infty} \dfrac{4n^2-n^3}{10+2n^3}$.

EXERCICE 2. Determine if the following series converge or diverge. If they converge give the value of the series.

(1) (a) $\sum_{n=1}^{\infty} 9^{2-n}4^{n+1}$, (b) $\sum_{n=0}^{\infty}(-4)^{3n}5^{1-n}$ $(c^{(*)})$ $\sum_n \dfrac{n^3+3n-1}{n!}$

(2) Use the results from the previous example to determine the value of the following series.

$$(c)\ \sum_{n=0}^{\infty} 9^{2-n}4^{n+1}, \qquad (d)\ \sum_{n=3}^{\infty} 9^{2-n}4^{n+1}$$

(3) **Telescoping Series :** Determine if the following series converges or diverges. If it converges find its value.

$$(e)\ \sum_{n=1}^{\infty} \frac{1}{n^2+3n+2}, \qquad (f^{(*)})\ \sum_{n=1}^{\infty} \frac{n!+2^n}{2^n(n)!}, \qquad (j)\ \sum_{n=1}^{\infty} \left(\frac{4}{n^2+4n+3} - 9^{2-n}4^{n+1}\right)$$

EXERCICE 3. **Integral, $p-$ series, Comparison and Limit comparison test** Determine if the following series are convergent or divergent.

$$(a)\ \sum_{n=2}^{\infty} \frac{1}{n\ln(n)}, \quad (b)\ \sum_{n=0}^{\infty} ne^{-n^2}, \quad (c)\ \sum_{n=4}^{\infty} \frac{1}{n^7}, \quad (d)\ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}, \quad (e)\ \sum_{n=1}^{\infty} \frac{n}{n^2-\cos^2(n)}, (f)\ \sum_{n=1}^{\infty} \frac{n!}{n^n},$$

$$(j^{(*)})\ \sum_{n=1}^{\infty} \frac{e^{-n}}{n+\cos^2(n)}, \quad (h)\ \sum_{n=1}^{\infty} \frac{\ln n}{n+\ln n}, \quad (i)\ \sum_{n=1}^{\infty} \frac{1+(-1)^n\sqrt{n}}{1+n}, \quad (g^{(*)})\ \sum_{n=1}^{\infty} \frac{n!}{3\times5\times7\times ...\times(2n+3)},$$

$$(k)\ \sum_{n=1}^{\infty} (1+1/n)^n/(n^2+1), \quad (l^{(*)})\ \sum_{n=2}^{\infty} \frac{1}{n\ln(n^2+1)}, \quad (m)\ \sum_{n=1}^{\infty} \frac{5^n-2^n}{7n+3n}, \quad (n^{(*)})\ \sum_{n=2}^{\infty} \frac{\cos^2 n}{n\ln^2 n}.$$

EXERCICE 4. **Alternating series and Absolute Convergence :**

(A)**:** Determine if the following series is convergent or divergent.

(1)

$$(a)\ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}, \qquad (b)\ \sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+5}, \quad (c^{(*)})\ \sum_{n=0}^{\infty} \frac{(-1)^{n-3}\sqrt{n}}{n+4}, \qquad (d)\ \sum_{n=2}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}.$$

(B)**:** Determine if each of the following series are absolute convergent, conditionally convergent or divergent.

$$(a)\ \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \qquad (b^{(*)})\ \sum_{n=1}^{\infty} \frac{(-1)^{n+2}}{n^2} \qquad (c)\ \sum_{n=1}^{\infty} \frac{\sin(n)}{n^3},$$

(C)**:** Let $u_n = \sin\left(\pi\left[\dfrac{n^3+1}{n^2+1}\right]\right)$. Show that : $\sum u_n$ is an alternating series, then determine its nature

(D)**:** Give the number required to approximate the sum of the following to within 2 decimal places(0.01)

$$(1)\ \sum \frac{(-1)^{n+1}}{n^2+1}, \quad (2)\ \sum \frac{n^2}{3^{n+1}}, \quad (3^{(*)})\ \sum \frac{1}{n^n}.$$