May 8, 1945 Guelma University **Department of Mathematics**

2nd year : licence Mathematics **Sheet.0 : Topology of $\mathbb{R}^n$, $(n = 2)$. Average time : two weeks**

EXERCICE 1.

(A): Determine and represent the domain of definition for the following functions

$$f_1(x,y) = \frac{\sqrt{x}y}{x^2+y^2}, \quad f_2(x,y) = \frac{\sqrt{x+y+1}}{x-1}, \; f_3(x,y) = x\ln(y^2-x), \quad (*)f_4(x,y) = \sqrt{4x-x^2+4y-y^2}.$$

(B): Represent the set A, then determine the interior A°, the boundary ∂A and the closure $\overline{A}$ in the following cases :

(1) $A = \left\{(x,y) \in \mathbb{R}^2 / \quad 0 < x < 1, \quad 1 \leq y \leq 3\right\}, \quad A = \left\{(x,y) \in \mathbb{R}^2 / \quad y \geq x^2, \quad |\, x \,| < 2\right\},$

(2) $A = \left\{(x,y) \in \mathbb{R}^2 / \quad x^2+y^2-2x+6y \leq 15\right\}.(*)$

EXERCICE 2.

(A): Draw and then examine whether A is an open or closed set in the following cases :

(1) $A = \left\{(x,y) \in \mathbb{R}^2 / \quad 3x^2+2y^2 < 6\right\}, \quad (3)(*)\; A = \left\{(x,y) \in \mathbb{R}^2 / \quad x^2+y^2 \geq \sqrt{x^2+y^2}\right\},$

(2) $A = \left\{(x,y) \in \mathbb{R}^2 / \quad \max\{|x|,|y|\} \leq 1\right\}, \quad (4)\; A = \left\{(x,y) \in \mathbb{R}^2 / \quad |x|+|y| < 1\right\}.$

(B): Find the Cartesian equation

(1) $0 \leq \varphi \leq \pi/2, \quad 0 \leq \rho \leq a\cos(\varphi),$

(2) $0 \leq \varphi \leq \pi/4, \quad 0 \leq \rho \leq a\cos(\varphi) + a\sin(\varphi).(*)$

(C): Rewrite the set A using polar coordinates

(1) $A = \left\{(x,y) \in \mathbb{R}^2 / \quad x \geq 0, \quad (x^2+y^2)^2 \geq x^2+y^2\right\},$

(2) $A = \left\{(x,y) \in \mathbb{R}^2 / \quad x > 0, \quad 1/2+y^2 \leq x^2 \leq 1-y^2\right\}.(*)$

EXERCICE .3. Sketch the sets of points below and indicate which ones are convex

(1) $A = \left\{(x,y) \in \mathbb{R}^2 / \quad -5 < y < -3x^2\right\},$

(2) $A = \left\{(x,y) \in \mathbb{R}^2 / \quad x^2+3y^2 > 2\right\}.$

(3) $A = \left\{(x,y) \in \mathbb{R}^2 / \quad y > -x^2\right\}.(*)$

(4) $A = \left\{(x,y) \in \mathbb{R}^2 / \quad x \geq 0, \quad y \leq 0\right\}.(*)$

B.R **Academic year 2023/2024.** $(*)$ **additional questions**