EXERCICE 1.

(1): Verify Fubini's theorem for the double integrals

$$I = \int\int_D \frac{dA}{\sqrt{2x-y}}, \quad D = [1,2] \times [0,1], \; J = \int\int_R \frac{xdA}{y^2}, \quad R = [1,2] \times [4,6].$$

Calculate I and J.

(2): Let D the region bounded by the curves $xy = 6$ and $x + y = 7$. Sketch the region D, then evaluate $\int\int_D (x+y)dxdy$.

(3): Let D the region bounded by the curves $x = y^2$ and $x = y^2/2 + 1$. Sketch the region D, then evaluate $\int\int_D (x-y)dxdy$.

(4): Sketch the region of integration for the integral $\int_0^2 \int_{x^2}^{2x} (4x+2)dydx$. Write an equivalent integral with the order of integration reversed.

(5): Evaluate $I = \int_{-\infty}^{+\infty} e^{-x^2} dx$.(Hint : $B(0,a) \subset R \subset B(0, \sqrt{2}a)$, where $R = [-a,a]^2$.)

EXERCICE 2.

(1) Find the area of the region R enclosed by the parabola $y = x^2$ and the line $y = x + 2$.

(2) Find the average value of $f(x,y) = x\cos(xy)$ over the rectangle $R: \; 0 \leq x \leq \pi, \; 0 \leq y \leq 1$.

(3) Find the limits of integration for integrating $f(r,\theta)$ over the region R that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$.

(4) evaluate $\int\int_R e^{x^2+y^2} dydx$, where R is the semicircular region bounded by the $x-$axis and the curve $y = \sqrt{1-x^2}$.

(5) Evaluate $\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx$, for $a > 0$ and $b > 0$.

(6) Show that $\frac{\pi}{3} \leq \int\int_D \frac{dA}{\sqrt{x^2 + (y-2)^2}} \leq \pi$, where D is the unit disc.

EXERCICE 3.

(1) Integrate $f(x,y,z) = z\sqrt{x^2+y^2}$ over the cylinder $x^2 + y^2 \leq 4$ for $1 \leq z \leq 5$.

(2) $(*)$ Integrate $f(x,y,z) = z$ over the cylinder $x^2 + y^2 \leq 4$ for $1 \leq z \leq y$.

(3) Let

$$\Delta_3 = \left\{(x,y,z) \in \mathbb{R}^3, \; x \geq 0, y \geq 0, z \geq 0 \quad x + y + z \leq 1\right\}.$$

Using a suitable change of variables, calculate the integral

$$\int\int\int_{\Delta_3} \frac{dxdydz}{(x+y+z+1)^3}.$$

(4) Let

$$G = \left\{(x,y,z) \in \mathbb{R}^3, \; x \geq 0, y \geq 0, z \geq 0 \quad x + z \leq 1, \; y + z \leq 1\right\}.$$

Draw the domain G. The domain G is it bounded ? Then calculate the triple integral

$$I = \int\int\int_G \frac{dxdydz}{(x+y+z)^3}.$$