EXERCICE 1.

A**:** Perform the following index shifts.

— Write $\sum_{n=1}^{\infty} ar^{n-1}$ as a series that starts at $n=0$. Write $\sum_{n=1}^{\infty} \frac{n^2}{1-3^{n+1}}$ as a series that starts at $n=3$.

— $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$, $\sum_{n=0}^{\infty}(-1)^n$ and $\sum_{n=0}^{\infty} \frac{4n^2-n^3}{10+2n^3}$.

EXERCICE 2. Determine if the following series converge or diverge. If they converge give the value of the series.

(1) (a) $\sum_{n=1}^{\infty} 9^{2-n}4^{n+1}$, (b) $\sum_{n=0}^{\infty}(-4)^{3n}5^{1-n}$

(2) Use the results from the previous example to determine the value of the following series.

$$(c) \sum_{n=0}^{\infty} 9^{2-n}4^{n+1}, \qquad (d) \sum_{n=3}^{\infty} 9^{2-n}4^{n+1}$$

(3) **Telescoping Series :** Determine if the following series converges or diverges. If it converges find its value.

$$(e) \sum_{n=0}^{\infty} \frac{1}{n^2+3n+2}, \qquad (f) \sum_{n=1}^{\infty} \frac{1}{n^2+4n+3},, \qquad (j) \sum_{n=1}^{\infty} \left(\frac{4}{n^2+4n+3} - 9^{2-n}4^{n+1}\right)$$

EXERCICE 3. **Integral, $p-$ series, Comparison and Limit comparison test**

(1) Determine if the following series are convergent or divergent.

$$(a) \sum_{n=2}^{\infty} \frac{1}{n\ln(n)}, \quad (b) \sum_{n=0}^{\infty} ne^{-n^2}, \quad (c) \sum_{n=4}^{\infty} \frac{1}{n^7}, \quad (d) \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}, \quad (e) \sum_{n=1}^{\infty} \frac{n}{n^2-\cos^2(n)},$$

(2)

$$(f) \sum_{n=1}^{\infty} \frac{e^{-n}}{n+\cos^2(n)}, \quad (j) \sum_{n=1}^{\infty} \frac{\ln n}{n+\ln n}, \quad (h) \sum_{n=1}^{\infty} \frac{1+(-1)^n\sqrt{n}}{1+n}, \quad (i) \sum_{n=1}^{\infty} \frac{n!}{3\times 5\times 7\times ...\times(2n+3)}.$$

EXERCICE 4. Determine if the following series converge or diverge.

$$(a) \sum_{n=0}^{\infty} \frac{1}{3^n-n}, \qquad (b) \sum_{n=2}^{\infty} \frac{4n^2+n}{\sqrt[3]{n^7+n^3}}.$$

EXERCICE 5. **Alternating series and Absolute Convergence :**

(A)**:** Determine if the following series is convergent or divergent.

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}, \qquad (b) \sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+5}, \quad (c) \sum_{n=0}^{\infty} \frac{(-1)^{n-3}\sqrt{n}}{n+4}, \qquad (d) \sum_{n=2}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}.$$

(B)**:** Determine if each of the following series are absolute convergent, conditionally convergent or divergent.

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \qquad (b) \sum_{n=1}^{\infty} \frac{(-1)^{n+2}}{n^2} \qquad (c) \sum_{n=1}^{\infty} \frac{\sin(n)}{n^3},$$

(C)**:** Let $u_n = \sin\left(\pi\left[\frac{n^3+1}{n^2+1}\right]\right)$. Show that : $\sum u_n$ is an alternating series, then

— Studying absolute convergence

EXERCICE 6. Determine if the following series are convergent or divergent.

(1) (a) $\sum_{n=1}^{\infty} \frac{(-10)^n}{4^{2n+1}(n+1)}$, (b) $\sum_{n=0}^{\infty} \frac{n!}{5^n}$, (c) $\sum_{n=2}^{\infty} \frac{n^2}{(2n-1)!}$,

(2) (d) $\sum_{n=1}^{\infty} \frac{9^n}{(-2)^{n+1}n}$, (e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$, (f) $\sum_{n=0}^{\infty} \frac{n+2}{2n+7}$.