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Polycopié de cours

Par

Dr. BELLAOUAR Djamel

**Anglais I et II
(Cours et exercices corrigés pour les
mathématiciens)**

La clé pour la prononciation correcte

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Les paroles s'envolent mais les écrits restent...

À cet effet ce polycopié!

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Introduction

English actually is the language of science, the language by which the world depends on the exchanging of ideas and knowledge. For this purpose, the content of this duplicated lecture notes includes the key for the correct pronunciation; by studying sounds of English or rather vowels and consonants. We also deal with the pronunciation problem of small and capital Greek letters used in Mathematics. In general, the manuscript is intended especially for students of mathematics and computer science at the university. This is the course of English I and II that I taught at University of 08 Mai 1945 Guelma for both Master and PHD students of the first year.

Mathematicians actually spend a great deal of time writing and improving their papers. The benefit of this presentation is to enable the student for future study by English and giving a guide to writing articles and notebooks easily and very well. It is also important for engineering, science and applied mathematics.

In general, in this manuscript, we state phrases used in pure and applied mathematics, some basic mathematical arguments and the correct pronunciation of certain mathematical expressions. Before discussing this, we have to look at the topic of phonetic symbols and grammar series.

In order to absorb the language to our students with its proper practice in the pronunciation, we provide a perfect dictionary for advanced learners of Mathematics, which contains the famous mathematical phrases in analysis, geometry, topology, algebra and number theory, differential calculus and many others. The dictionary includes the essential notions on general mathematics such as: numbers, sequences, functions, limits and continuity, derivatives, integrals, partial derivatives, vectors, applications of partial derivatives, multiple integrals, line integrals, surface integrals and integral theorems, infinite series, improper integrals, Fourier series, Gamma and Beta functions, functions of a complex variable and Fourier integrals.

We shall explore a number of applications of special phrases and sentences which are used in mathematical papers. This chapter is separate from other chapters because it is written using Latex and its numbering is also independent.

At the end, we finish this manuscript by providing the previous my exams and their solutions which carried at University of 08 Mai 1945 Guelma, from 2012 to 2016.

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Belaouar Djamel. February, 2019.

Mathematical English Dictionary

with

Phonetic Symbols

For beginners

by

Dr. Bellaouar Djamel

bellaouar.djamel@univ-guelma.dz, bellaouardj@yahoo.fr

- Helps you learn the most important mathematical words by English and French and how to use them.
- Helps you learn the phonetic symbols of some Mathematical phrases
- The Dictionary includes the following subfields:

Analysis / ə'næləsɪs /

Algebra / 'ældʒɪbrə /

Geometry / dʒɪ'ɒmɪtri /

Functional Analysis / 'fʌŋkʃnəl / ə'næləsɪs /

Numerical Analysis / nju:'merɪkəl ə'næləsɪs /

Probability / ,prɒbə'bɪlɪti /

LEVEL 1

Part 1. Mathematical English Dictionary

1.1. Sounds of English; Vowels and Consonants

Sounds of English

VOWELS

ɪ	ʊ	ʌ	ɒ	ə	e	æ		'short'
i:	u:	a:	ɔ:	ɜ:				'long'
ɪə	ʊə	aɪ	ɔɪ	əʊ	eə	aʊ	eɪ	diphthongs

CONSONANTS

p	t	tʃ	k	f	θ	s	ʃ	voiceless
b	d	dʒ	g	v	ð	z	ʒ	voiced
m	n	ŋ	h	l	r	w	j	

bbclearningenglish.com

1.2. Vowels ['vaʊəlz]

ə	i:	ɪ	æ	e	ʌ	
ɔ:	ɒ	a:	u:	ʊ	ə:	
eɪ	eə	aɪ	ɔɪ	aʊ	ɪə	əʊ

1.3. Consonants ['kɒnsənənts]

s	z	ʃ	ʒ	tʃ	dʒ	f	v
θ	ð	p	b	t	d	k	g
l	r	j	w	ŋ	n	m	h

1.4. Alphabet Letters with phonetic symbols

In mathematical presentation the correct pronunciation of letters using indices and powers is very important. For example, the expression $\frac{p_i}{q}$ pronounces:

pi: aɪ 'æʊvəʳ kju:

Letters ['letə(r)z]

<i>a</i> [eɪ]	<i>J</i> [dʒeɪ]	<i>S</i> [es]
<i>b</i> [bi:]	<i>k</i> [keɪ]	<i>t</i> [ti:]
<i>c</i> [si:]	<i>l</i> [el]	<i>U</i> [ju:]
<i>d</i> [di:]	<i>M</i> [em]	<i>v</i> [vi:]
<i>e</i> [i:]	<i>n</i> [en]	<i>W</i> ['dʌblju:]
<i>f</i> [ef]	<i>O</i> [əʊ]	<i>X</i> [eks]
<i>g</i> [dʒi:]	<i>P</i> [pi:]	<i>y</i> [waɪ]
<i>h</i> [eɪtʃ]	<i>Q</i> [kju:]	<i>Z</i> [zed], [zi:]
<i>i</i> [aɪ]	<i>R</i> [ɑ:(r)]	

1.5. Some words with phonetic symbols

word	[wɜ:d]	wife	[waɪf]
arm	[ɑ:m]	substitute	['sʌbstɪtju:t]
question	['kwɛstʃən]	problem	['prɒbləm]
sister	['sɪstəʳ]	water	['wɔ:təʳ]
party	['pɑ:tɪ]	try	[traɪ]
future	['fju:tʃəʳ]	quadrature	['kwɒdrətʃəʳ]
Baby	['beɪbɪ]	dangerous	['deɪndʒərəs]
substitution	[,sʌbstɪ'tju:ʃən]	translation	[trænz'leɪʃən]
translate	[trænz'leɪt]	transpose	[træns'pəʊz]
book	[bʊk]	France	[frɑ:ns]
child	[tʃaɪld]	children	['tʃɪldrən]
smile	[smaɪl]	cucumber	['kju:kʌmbəʳ]
important	[ɪm'pɔ:tənt]	satisfy	['sætɪsfaɪ]
situation	[,sɪtʃə'eɪʃən]	point	[pɔɪnt]
picture	['pɪktʃəʳ]	south]	[saʊθ
wild	[waɪld]	literature	['lɪtərɪtʃəʳ]

1.6. Small Greek letters used in Mathematics

Lower case Greek alphabet					
name	symbol	name	symbol	name	symbol
alpha	α	iota	ι	rho	ρ
beta	β	kappa	κ	sigma	σ
gamma	γ	lambda	λ	tau	τ
delta	δ	mu	μ	upsilon	υ
epsilon	ϵ	nu	ν	phi	ϕ
zeta	ζ	xi	ξ	chi	χ
eta	η	omicron	o	psi	ψ
theta	θ	pi	π	omega	ω

1.7. On the correct pronunciation of Greek Alphabets

alpha ['ælfə]	iota [aɪ'əʊtə]	Rho ['rəʊ]
beta ['bɪtə]	kappa	sigma [sɪgmə]
gamma ['gæmə]	lambda	tau [tə:]
delta ['deltə]	mu [mjʊ:]	upsilon ['ʌpsɪ,lɒn]
epsilon [epsɪlən]	nu [nju:]	phi [faɪ]
zeta ['zɪtə]	xi [zaɪ]	chi [kaɪ]
eta ['ɪtə]	omicron [əʊ'maɪkrɒn]	psi ['psɪ]
Theta ['θi:tə]	pi [paɪ]	omega ['əʊmɪgə]

and alos, we have

α	alpha	β	beta	γ	gamma	δ	delta
ϵ, ε	epsilon	ζ	zeta	η	eta	θ, ϑ	theta
ι	iota	κ	kappa	λ	lambda	μ	mu
ν	nu	ξ	xi	o	omicron	π, ϖ	pi
ρ, ϱ	rho	σ	sigma	τ	tau	υ	upsilon
ϕ, φ	phi	χ	chi	ψ	psi	ω	omega

1.8. Capital Greek letters used in Mathematics

B	Beta	Γ	Gamma	Δ	Delta	Θ	Theta
Λ	Lambda	Ξ	Xi	Π	Pi	Σ	Sigma
Υ	Upsilon	Φ	Phi	Ψ	Psi	Ω	Omega

1.9. Alphabetical English Dictionary of Mathematics

In this section we present a simple dictionary which contains the famous mathematical words and phrases. These words are used in elementary and advanced mathematics. Readers unfamiliar phonetic symbols are referred to the dictionary [3-4].

A

A set equipped with a distance, *un ensemble muni par une distance*

Abel ['eɪbəl], *Abel*^m

Abelian [ə'bi:liən] adjective, *abélien*^{adj}

Abelian group, *groupe abélien (commutatif)*

Abelian law, *loi commutative*

Above [ə'boʊv], *au-dessus*

Absolute ['æbsəlu:t], *absolu(e)*

Absolute value, *valeur absolue.*

Absolutely [ˌæbsə'lu:tli], *absolument, absolument convergente (intégrale, série)*

absolutely convergent series *série absolument convergente*

Acknowledgements [ək'nɒlɪdʒmənts]

Add [æd], *ajouter*

Additionally [ə'dɪfɪnəlɪ] *adverb en outre, de plus*

Admit [əd'mɪt], *admettre*

Algebra : the branch of mathematics that deals with variables or unknowns representing the arithmetic numbers

a.e. almost everywhere, *p.p presque partout*

Algebra ['ældʒɪbrə], *algèbre*

Algebraic [ˌældʒɪ'breɪk] *adj*, *algébrique* *adj*

Algebraic multiplicity, algebraic structure, algebraic and topological structure

Algorithm : A rule or procedure used to solve a mathematical problem

Algorithm ['ælgə,rɪðəm], *algorithmie*

all [ɔ:l]

Analogous *adj* [ə'næləgəs], *analogue* *adj*

Analysis [ə'næləsɪs], pl **analyses** [ə'nælɪsɪz], *analyse* *f*

analytic, analytical *adjective* [ˌænə'lɪtɪkəl] *analytique*

Answer ['ɑ:nsə], *réponse* *f*, *solution* *f*

Antisymmetric [ˌæntɪsɪ'metrɪk], *antisymétrique* *adj*

appendix [ə'pendɪks] **appendixes** or **appendices** of book
appendice *m* of document *annexe* *f*

applicable [ə'plɪkəbl], *applicable* (to : à)

Application [ˌæplɪ'keɪʃən], *application* *f*

Applied [ə'plaɪd], *appliqué*

Applied Linear Algebra, *algèbre linéaire appliquée*

Appreciable [ə'pri:ʃəbl]

Appreciable, *appréciable* *ad*

Approach [ə'prəʊtʃ], approach value, *valeur approchée* *adj*

Approximation [ə,prɒksɪ'meɪʃən], *approximation* *f*

arbitrarily ['ɑ:brɪtrəri], *arbitrairement*

arbitrarily close to *arbitrairement proche de*

Arbitrary ['ɑːbrɪtərɪ] adj, *arbitraire*^{adj}

arc [ɑːk], *arc*^m

arc sine x

Area ['ɛərɪə] *domaine*^m,

Argument ['ɑːgjʊmənt], *argument*

Argument, the argument of a complex number

Arithmetic [ə'riθmətɪk], *arithmétique*

Article ['ɑːtɪkl], *article*^m

assembly [ə'sembli]

Assertion [ə'sɜːʃən], **statement**, *affirmation*^m, *assertion*^m

Associative [ə'səʊʃɪətɪv], *Mathematics*, *associatif-ive*

Associativity, *associativité*, *L'associativité de l'addition dans \mathbb{R}* .

Assume [ə'sjuːm], *supposer*, *supposons que*,

Assumption [ə'sʌmpʃən], *hypothèse*^m

Asymptotic, *asymptotique*

attention [ə'tenʃən]

Automorphism, *automorphisme*^m [ɔtɒmɔːfɪzəm]

average ['ævərɪdʒ], *moyenne*^f

Duplicated lecture notes, *polycopié*^m

Axiom : a statement regarded as self-evident; accepted without proof

Axiom ['æksɪəm], *axiome*^m

axis ['æksɪs] *noun*, **pl axes**, *axe*^m

B

Ball [bɔ:l], *boule*^f

Bar [bɑ:r], *barre*^f, we say X bar, *On dit X barre.*

Basic ['beɪsɪk], *fondamental*^{adj}, *essentiel*^{adj}, *élémentaire*^{adj}

Basis ['beɪsɪs] pl **bases**, *base*^f

because [bi'kɒz]

Because, since [bi'kɒz], *puisque, car, comme*

behaviour, behavior US [bi'heɪvjər]

being ['bi:ɪŋ]

Belong [bi'lɒŋ], *appartenir à*

below [bi'ləʊ]

Best [best], *le meilleur, la meilleure*

Best approximation, *la meilleure approximation*

Bibliography [ˌbɪblɪ'ɒgrəfɪ], *bibliographie*^f, *référence*^f

Bijjective [baɪ'dʒektɪv], *bijectif*^{adj}

Bijjective function

Bilinear, *bilinéaire*, ♦ **Math.** *Application, forme bilinéaire pour un couple de variables, linéaire par rapport aux deux variables.*

Binary ['baɪnəri], *binaire*

Binary relation, *relation binaire*

Binomial : an expression with two terms

Binomial [baɪ'nəʊmɪəl], *Mathematics*, *binôme*^m

Bisection [baɪ'sekʃən], *division en deux parties égales, bissection*^f

Bnach space, *un espace de Banach*

Body ['bɒdɪ], *Field, corps*^m

Bound, [baʊnd], **bounds**, [baʊndz], *limite(s)*^{f(pl)}, *bornes*

Boundary ['baʊndərɪ], *limite*^f, *frontière*^f

Bounded ['baʊndɪd]

bounded above, **bounded below**, *borné(e) supérieurement, borné(e) inférieurement*

Boundless ['baʊndlɪs], *infini, illimité*

bracket ['brækɪt], **bracket** ['brækɪt], *parenthèse*^f left bracket *parenthèse à gauche* right bracket *parenthèse à droite*

Branch [brɑːntʃ], *branche*^f

By using the ..., *En utilisant ...*



Calculate ['kælkjʊleɪt], *calculer*^v

Calculus, pl **calculuses** ['kælkjʊləs], *calcul*^m

Canonical [kə'nɒnɪkəl], *canonique*^{adj}

Cardinal ['kɑːdɪnəl], *adjective, cardinal*

Cartesian [kɑː'tiːziən] *adjective ; noun cartésien(ne)*^{m(f)}

Cartesian coordinates *plural noun Mathematics : coordonnées*^{fp} *cartésiennes*

category ['kætɪgərɪ], *catégorie*^f

Centre, center US ['sentər], *centre*^m

certain ['sɜ:tən]

chain [tʃeɪn], *chaîne*^f

Change of basis *changement de base*

changing [tʃeɪndʒɪŋ] *adjective* *variable, changeant*

Chapter [tʃæptə], *chapitre*^m

character ['kærɪktə] *noun* *caractère*^m

characteristic [kærɪktə'rɪstɪk], *caractéristique*^{adj}

characteristic polynomial

characterization [kærɪktərəɪ'zeɪʃən], *interprétation*, *caractérisation*^f

characterize ['kærɪktəraɪz]

choice [tʃɔɪs]

circle ['sɜ:kəl], *cercle*^m

close [kləʊs], *proche*

Closed [kləʊzd], *fermé*^{adj}

Closure ['kləʊʒə], *fermeture*^f

coefficient [kəʊɪ'fɪʃənt], *coefficient*^m

cofactor ['kəʊfæktə], *comatrice, cofacteur*^m

Collection [kə'lekʃən], *collection*^f

Column ['kɒləm], *colonne*^f *column vector*, *vecteur colonne*

Combination [kəm'bɪ'neɪʃən], *combinaison*^f

combinatorial, combinatory *Mathématique combinatoire*
combinatorial analysis, combinatorics, Mathematical Induction in
Combinatorics

comma ['kɒmə], *virgule*^f

comment ['kɒment], *commentaire*^m, *remarque*^f

Comments about the chapter II, *commentaires sur le chapitre II*

common ['kɒmən]

Commutative^{adj} [kə'mju:tətɪv], *lois*^{fp} *commutatives*

Commutativity, commutative property

Compact [kəm'pækt], *compact*^{adj}

Compact self-adjoint operators on a Hilbert space

compactness [kəm'pæktɪs] *noun* *compacité*^f

compactness [kəm'pæktɪs] *noun* *compacité*^f

Comparable ['kɒmpərəəbl], *comparable*

Comparison [kəm'pærɪsn], Comparison test, *comparaison*^f

Complete [kəm'pli:t], *complet* (-ète)^f, *un espace complet*

Complex ['kɒmpleks], *complexe*^{adj}

complex-valued function *fonction à valeurs complexes*

Component [kəm'pəʊnənt], *composant*

Components [kəm'pəʊnənts], *les composants de X*

Composite ['kɒmpəzɪt], *Mathematics*, *composé*

Composite number, not prime, *nombre composé*

composition [kɒmpə'zɪʃən] *composition*^f

computation [ˌkɒmpjʊ'teɪʃən] noun *calcul*^m *estimation*^f, *évaluation*^f

Compute [kəm'pjʊt], *Calculer*

Concept ['kɒnsept], *notion*^f, *idée*^f, *concept*^m

Conclusion [kən'kluːʒən], *conclusion*^f, *fin*^f

Condition [kən'dɪʃən], *condition*^f

conditional [kən'dɪʃənl], *conditionnel*

cone [kəʊn] noun *Mathematics*, *cône*^m

congruence ['kɒŋgrʊəns], *Mathematics*, *congruence*^f

Conjecture [kən'dʒektʃər], *conjecture*^f

Conjugate ['kɒndʒʊgeɪt], *conjuguée (matrice)*

Conjugate or Dual of an Operator

Connected [kə'nektɪd] adj connected and disconnected
Mathematics, connected space, *connexe*^{adj}

consequence ['kɒnsɪkwəns] noun *conséquence*^f,

consequence ['kɒnsɪkwəns] noun, *conséquence*^f

Constant ['kɒnstənt], *constante*^f, *un nombre constant*^{adj}

constant *constant(e)*^{adj}

constant function, *fonction constant(e)*

construction [kən'strʌkʃən], *construction*^f,

Contained [kən'teɪnd], contained in A.

Containing A

Continuous [kən'tɪnjʊəs], *continu(e)*

Contraction [kən'trækʃən], *contraction*^f

Contradiction [,kɒntrə'dɪkʃən], *contradiction*^f

convention [kən'venʃən] **noun** *convention*^f *by convention $0! = 1$ and $a^0 = 1$, *par convention $0! = 1$ et $a^0 = 1$. By convention, the degree of $p=0$ is $-\infty$.**

Converge [kən'vɜːdʒ], *converger*^v

Convergence [kən'vɜːdʒəns], *convergence*^f

Convergence and Continuity

Convergent [kən'vɜːdʒənt], *convergent(e)*^{adj}

Converse ['kɒnvɜːs], **inverse**

Conversely [kɒn'vɜːslɪ], *inversion*

Convex ['kɒn'veks], *convexe*

Coordinate [kəʊ'ɔːdɪnɪt], *Mathematics*, *coordonnée*

Corollary [kə'rɒləri], *corollaire*^m

Cosine ['kɒsɪn], *cosinus*^m

Countable ['kaʊntəbl] *adjective*, *dénombrable*

Countable dense subset, *sous-ensemble dense dénombrable*

counterexample ['kaʊntərɪg,zɑːmpəl], **noun**, *contre-exemple*^m

counting ['kaʊntɪŋ], *calcul*^m, the prime counting function

Couple ['kʌpl], *couple*^m

Course [kɔːs], *cours* **nom masculin**

Criterion [kraɪ'tɪəriən] **noun**, **pl** **criteria** or **criteria**
[kraɪ'tɪəriə], *critère*^m

cryptography [krɪp'tɒgrəfɪ] **noun** *cryptographie*^f

cube [kjuːb], *Mathematics*, *cube*^m

Cubic ['kjuːbɪk], *cubique*^{adj}

curve [kɜːv], *courbe*^f

cyclic, ['saɪklɪkəl], **cyclical**, *adjective*, *cyclique*

D

Decomposition [ˌdiːkɒmpə'zɪʃən], *décomposition*^f

Decreasing [diː'kriːsɪŋ], *décroissant*^{adj}

Define [dɪ'faɪn], *définer*, *on définit*

Definite ['defɪnɪt], *défini-e*^{adj}

Definite integral,

definitely ['defɪnɪtli]

Definition [ˌdefɪ'nɪʃən], *définition*^f

Definitions and basic properties

degenerate *dégénéré(e)*

Degree [dɪ'ɡriː], *degré*^m

Demonstrable ['demənstreɪbl] *démontrable*^{adj}

Demonstration [ˌdemən'streɪʃən], *démonstration*^f

Denominator [dɪ'nɒmɪneɪtər], *dénominateur*^m

Denote [dɪ'nəʊt], *indiquer*, *dénoter*, *on note*

Dense [dens], *dense*^{adj}

Density ['densɪtɪ], *densité*^f

Department [dɪ'pɑːtmənt] **noun** *département*^m

depend on *dépendre de*

derivation [ˌderɪ'veɪʃən], *dérivation*^f

Derivative [dɪ'rɪvətɪv], **Mathematics**, *dérivée*^f

Determinant [dɪ'tɜːmɪnənt], *déterminant*^m

Development [dɪ'veləpmənt], *développement*^m

diagonal [daɪ'æɡənəl], *diagonal*^{adj}, *diagonale*^{nom,adj}

Diagonalizable, *diagonalizable*^{adj}

Different ['dɪfrənt], (different from) not the same, *différent*

differentiable function *fonction dérivable*

differential [ˌdɪfə'renʃəl], *différentiel*, *différentielle*

Differential equation, *équation différentielle*

Differential geometry, *géométrie différentielle*

Differential operator

Differentiation [ˌdɪfərənʃɪ'eɪʃən], **Mathematics** *différentiation*^f

Digit ['dɪdʒɪt] **noun** **Mathematics**, *chiffre*^m

Dimension [daɪ'menʃən], *dimension*^f

Direct [daɪ'rekt], *direct-e*, **(direct) sum** *somme (directe)*

Direct sum of subspaces, **Direct sums**

directly [dɪ'rektlɪ] **adverb** = straight *directement*

Disconnected [ˌdɪskə'nektɪd] **adjective**

discrete [dɪs'krɪt] adjective Mathematics, *discret* (-ète)^f

discriminant [dɪs'krɪmɪnənt], Mathematics *discriminant*^m

discussion [dɪs'kʌʃən] noun *discussion*^f, *débat*^m

Disjoint [dɪs'dʒɔɪnt] adjective Mathematics, *disjoint*

Disjoint sets, *ensembles disjoints*

Distance ['dɪstəns], *distance*^f

distinction [dɪs'tɪŋkʃən] noun = difference *distinction*^f,

Distribution [dɪstrɪ'bjuːʃən], *distribution*^f

Distributions and Sobolev Spaces

Diverge [daɪ'vɜːdʒ], *diverger*^v

Divergence [daɪ'vɜːdʒəns], noun, *divergence*^f

Divergent, adjective [daɪ'vɜːdʒənt]

Divided [dɪ'vaɪdɪd], *divisé*

Divisibility, *la divisibilité*^f

Divisible [dɪ'vɪzəbl], *divisible*^{adj} (by : *par*)

Division [dɪ'vɪʒən], *la division*

divisor [dɪ'vaɪzəʳ] noun Mathematics *diviseur*^m

Domain [dəʊ'meɪn], *domaine*^m

dominant ['dɒmɪnənt] *dominant*

Dot [dɒt], **pois**^m Mathematics, *point*^m

double ['dʌbl] adjective *double*

Double ['dʌbl], *double*^{adj}

Dual ['djʊəl], *duel*^m

E

easily ['i:zili] adverb *facilement*

Easy ['i:zi], *facile*^{adj}, *simple*^{adj}

Eigenspace, *espace propre*

Eigenvalue, ['i:gæn 'vælju:], *valeur propre*

Eigenvalues and eigenvectors of a symmetric matrix

Eigenvector, *vecteur propre*

Element ['elɪmənt], *élément*^m

elementary [ˌelɪ'mentəri]

Elementary Number Theory, is the purest branch of pure mathematics.

Elements of Hilbert Space

Empty ['emptɪ], *vide*, the set with no elements. *L'ensemble vide*.

end [end]

Endomorphism [ˌendəʊ'mɔ:fiʒəm], *endomorphisme*^m

entire [ɪn'taɪər] adjective (*tout*) *entier* before plural noun
entier

Epsilon, *epsilon*, [ɛpsɪlən]

Equal ['i:kwəl], *Mathematics*, *égal*

Equality [ɪ'kwɒlɪti], *égalité*^f

Equation [ɪ'kweɪʒən], *Mathematics*, *Chemistry*, *équation*^f

Equipped [ɪ'kwɪpt], *muni-e*,

Equivalence [ɪ'kwɪvələns], *équivalence*

Equivalence relation

Equivalent [ɪ'kwɪvələnt], *adjective* *équivalent*

establish [ɪs'tæblɪʃ]

estimate ['estɪmət] *estimation*^f

etc [ɪt'setərə], abbreviation of **et cetera** : abréviation de *et cætera*, *etc*

Euclid's Algorithm

Euclidean [ju:'klɪdɪən], *euclidien*, non-Euclidean geometry, *géométrie*^f *non-euclidienne*

Evaluate [ɪ'væljʊeɪt], *évaluer*, *calculer*

evaluation [ɪ,væljʊ'eɪʃən], *évaluation*^f

Even ['i:vən], *pair*, *paire*^{adj}

Even function, *fonction paire*

Every ['evrɪ], for every, *tout*, *chaque*, *tous*, *pour tout*

Evident ['evɪdənt], *évident*^{adj}

Exact [ɪg'zækt], *solution exacte*

Example [ɪg'zɑ:mpəl], *exemple*^m

Except [ɪk'sept], *sauf*

Exercise ['eksəsaɪz], *exercice*^m

Existence [ɪg'zɪstəns], *existence*^f

Expansion [ɪk'spænjən], *développement*^m

explain [ɪk'spleɪn] *verb* *expliquer*

exponent [ɪk'spəʊnənt] **noun**, Mathematics, *exposant*^m

Exponential [ˌɛkspəʊ'nɛnʃəl], *exponentiel*

Exponentiation

express [ɪk'spres]

Expression [ɪk'spreʃən], *expression*^f

Extension [ɪk'stɛnʃən], *extension*^f

External [ɪk'stɜːnl] *externe*

F

Factor ['fæktər], Mathematics, *facteur*^m, *élément*^m

Factorial [fæk'tɔːriəl], *factoriel*

Factorization, *factorisation*

Factorize ['fæktəraɪz], Mathematics, *mettre en facteurs*

False [fɔːls] *faux, fausse*

Family ['fæmɪli], *famille*^f

Famous ['feɪməs] *célèbre*

Fibonacci sequence [ˌfɪbə'nɑːtʃɪ'sɪkwəns], **Fibonacci series** [ˌfɪbə'nɑːtʃɪ'sɪəriːs] **noun** Mathematics *suite*^f *de Fibonacci*

Field [fiːld], *corps*

finally ['faɪnəli] **adverb** *finalement*

Find [faɪnd], *trouver*, **we find**, *on trouve*

Finite ['faɪnaɪt], *limité, fini, finie*

finite dimension, *dimension finie*

infinite dimension, *dimension infinie*

Finite dimensional, *de dimension finie*

finite element method

finite set, *ensemble fini*

First [fɜːst], *premier*

First order differential equations, *équations différentielles du premier ordre.*

Firstly [ˈfɜːstli] **adverb**, *d'abord, premièrement*

Fixed [fɪkst], unique fixed point, *point fixe unique*

Following [ˈfɒləʊɪŋ], *suivant, suivante*

For all, *pour tout*, **For every**, *pour tout*

Form [fɔːm], *forme*^f

Formula [ˈfɔːmjʊlə] pl formulas [ˈfɔːmjʊləs] or formulae [ˈfɔːmjʊliː], *formule*

formulation [ˌfɔːmjʊˈleɪʃən] **noun** *formulation*^f

Fraction [ˈfrækʃən], **Mathematics**, *fraction*^f

Free [friː], *libre*

From the hypothesis, *d'après l'hypothèse*

Function [ˈfʌŋkʃən], *fonction*^f **Math.** Relation qui existe entre deux quantités, telle que toute variation de la première entraîne une variation correspondante de la seconde (ou en terme d'ensembles, étant donné deux ensembles X et Y, toute opération qui associe à tout élément x de X un élément y de Y que l'on note $f(x)$).

function in three variables *fonction en trois variables*

Functional [ˈfʌŋkʃnəl], *fonctionnel, analyse fonctionnelle*

Functional analysis, *analyse fonctionnelle*

Fundamental [ˌfʌndə'mentl], *fondamental, essentiel*

fuzzy ['fʌzi], *flou*



gcd, **The greatest common divisor**. *Le p.g.c.d, le plus grand commun diviseur*

General ['dʒenərəl], *général*

generalization [ˌdʒenərələɪ'zeɪʃən], *généralisation*^f

geometric series *série géométrique*

Geometry [dʒɪ'ɒmɪtri], *géométrie*^f

global ['glɔːbl] *adjective*

global maximum *maximum global*

local maximum *maximum local*

global minimum *minimum global*

local minimum *minimum local*

Graph [grɑːf], *graphe*^m

graphic ['græfɪk], *graphique*^{adj}

Group [gruːp], *groupe*^m



half-open interval *intervalle demi ouvert*

Harmonic [hɑː'mɒnɪk], *Mathematics, harmonique*

Heat [hiːt], *chaleur*^f

Heat equation, *équation de la chaleur*.

Hence [hens], *d'où*

High [haɪ], *haut*, higher dimensions

Hilbert Spaces

hint [hɪnt], hint of the proof

hold [həʊld], **holds** [həʊldz]

Homeomorphism *homéomorphisme*^m [ɔmeɔmɔrfɪsm]

Homogeneous [ˌhəʊməˈdʒiːniəs], *homogène*

Homogeneous system, *système homogène*

homomorphism [ˌhɒməˈmɔːfɪzəm] **noun**

hyperbolic [ˌhaɪpəˈbɒlɪk], **hyperbolical** [ˌhaɪpəˈbɒlɪkəl],
hyperbolique

Hyperbolic function, **Mathematics** : *fonction*^f *hyperbolique*.

Hypothesis [ˌhaɪˈpɒθɪsɪs] **noun**, **pl hypotheses** [haɪˈpɒθɪsɪz],
hypothèse^f

I

i-th column

i.e., identically equivalent, *identiquement équivalente*

Idea [aɪˈdɪə], *idée*^f

ideal [aɪˈdɪəl], **adjective or noun**, *idéal*^m

identically [aɪˈdentɪkəlɪ]

Identity [aɪˈdentɪtɪ], *identité*^f

Identity matrix, Identity map

If and only if, *si et seulement, si*

iff ['If], if and only if, *si et seulement si*

illustrate ['Iləstreɪt]

illustration [ˌɪləs'treɪʃən] **noun** *illustration*^f

Image ['ɪmɪdʒ], *image*^f

Imaginary [ɪ'mædʒɪnəri], *imaginaire*^{adj}

Imaginary number (**Mathematics**) : *nombre*^m *imaginaire*

implication [ˌɪmplɪ'keɪʃən], *implication*^f

Implies that, *implique*

important [ɪm'pɔ:tənt], *important-e*

Important, *the most important concept concerning sequences is convergence.*

Improper [ɪm'prɒpə], *improper*

improve [ɪm'pru:v], *améliorer, développer*

improvement [ɪm'pru:vmənt]

in other words *autrement dit*

Increasing [ɪn'kri:ɪŋ], *croissant, suite croissante*

Indeed [ɪn'di:d], *en effet*

Indefinite [ɪn'defɪnɪt], *indéfini-ie, illimité*

Indefinite integral

independence [ˌɪndɪ'pendəns], *indépendance*^f

independent [ˌɪndɪ'pendənt], *indépendant*

Indeterminate [ˌɪndɪ'tɜːmɪnɪt]

indeterminate form of type zero over zero

index [ˈɪndeks] pl **indices** [ˈɪndɪsɪz], *indice*^m

Induction [ɪn'dʌkʃən], *réurrence*

inequality [ˌɪni'kwɒlɪtɪ], *inégalité*^f

inferior [ɪn'fɪərɪər]

Infinite [ˈɪnfɪnɪt], *infini*, *illimité*^{adj}

Infinite dimensional, *de dimension infinie*

Infinitely [ˈɪnfɪnɪtli], *infiniment*

Infinitesimal [ˌɪnfɪnɪ'tesɪmə], *Mathematics infinitésimal*^{adj}

Infinity [ɪn'fɪnɪtɪ], *infinité*^f, *infini*^m

Infinity, the limit of f as x tends to infinity is a , *la limite de f lorsque x tend vers l'infini est a .*

Initial [ɪ'nɪʃəl], *initial*^{adj}

Initial condition, *condition initiale*

Initial value, *valeur initiale*

Injective *injective*

Inner [ˈɪnər], **inner product**, *produit scalaire.*

Inner product spaces, *espaces préhilbertiens*

Integer [ˈɪntɪdʒər], *entier (nombre*^m)

Integrable, *intégrable*^{adj}

Integral [ˈɪntɪgrəl], *intégral*

Integral operator

Integration [ˌɪntɪˈgreɪʃən], *intégration*^f

Interior [ɪnˈtɪəriəʳ], *intérieur* (-eure^f)

Internal [ɪnˈtɜːnl], *interne*

Interpolation [ɪnˌtɜːpəˈleɪʃən], *interpolation*^f

Intersection [ˌɪntəˈsekʃən] *Mathematics*, *intersection*^f

Interval [ˈɪntəvəl], *intervalle*^m

Introduce [ˌɪntrəˈdjuːs] *présenter*

introduction [ˌɪntrəˈdʌkʃən], *introduction*^f

Inverse [ˈɪnvɜːs], *inverse*

Invertible matrix, *matrice inversible*

Invertible, invertible matrices, *inversible*^{adj}

involve [ɪnˈvɒlv] **involving** *faisant intervenir*

Irrational [ɪˈræʃən], *Mathematics*, *irrationnel*^{adj}

irreducible [ˌɪrɪˈdjuːsəbl], *irréductible*^{adj}

irregular [ɪˈregjʊləʳ] *adjective*, *Mathematics*, *irrégulier*

isometric [ˌaɪsəʊˈmetrɪk], *isométrique*

isomorphism [ˌaɪsəʊˈmɔːfɪzəm] *noun*, *isomorphisme*^m

It follows that, *il vient*

Iterate, *itérer*

Iterative [ˈɪtəətɪv], *itératif*, *itérative*^{adj}

Iterative Methods for Solving Linear Systems



Jacobi's method, *Méthode de Jacobi*

Jacobian *le jacobien* [= le déterminant de la matrice jacobienne]

Jacobian matrix, *matrice jacobienne*

Kernel ['kɜ:nl], *noyau*^m

L

L.H. S. [= **left hand side**] *terme de gauche*

Laboratory [lə'brɒrətərɪ], *laboratoire*^m

Large [laɪdʒ], *grand*

Large enough *assez grand*

sufficiently large *suffisamment grand*

Law [lɔ:l], *loi*^f

Leading ['li:dɪŋ], the leading coefficient

Least [li:st], *le plus petit, la plus petite*. **Least squares method**,

Least upper bound of a set

Lemma ['lemə] noun, pl **lemmas** or **lemmata** ['lemətə], *lemme*

Let f be a function, *Soit f une fonction*

Let [let], let E be a nonempty set, *Soit E un ensemble non vide*.

likewise ['laɪkwaɪz] **adverb** *de même, également, aussi, de plus, en outre*

Limit ['lɪmɪt], *la limite*^f

Limited ['lɪmɪtɪd], *limité, borné*^{adj}

Line [laɪn], Mathematics, *ligne*^f

Linear ['lɪnɪə], *linéaire*^{adj}

Linear Algebra

Linear Operator, linear maps, linear equation, ...

Linearly dependent

linearly dependent, *liés, linéairement dépendants*

Linearly independent, *libres, linéairement indépendants*

Log [lɒg]. *log* **logarithme**^m

Logarithm ['lɒgərɪθəm] *logarithme*^m

Logic ['lɒdʒɪk], *logique*^f

Lower ['ləʊə], *inférieur* (-eure)^f

Lower bound

Lower triangular matrix, *matrice triangulaire inférieure*

LU factorisation

M

$m \times n$ **matrix** [*m by n matrix*], *matrice à m lignes et n colonnes*

Manner ['mænə], *manière*^f, *façon*^f

Map ['mæp], *Mathematics*, *application*^f

Maple ['meɪpl]

Mapping ['mæpɪŋ], *Mathematics*, *application*^f

Maps and their graphs

Mathematical [ˌmæθə'mætɪkəl], *mathématique*^{adj}

Mathematical induction,

mathematically [ˌmæθəˈmætɪkəlɪ] **adverb** in general
mathématiquement

Mathematician [ˌmæθəməˈtɪʃən], **noun** *mathématicien(ne)^{m(f)}*
mathématicien(ne)^{m(f)}

Mathematics [ˌmæθəˈmætɪks], **noun** *mathématiques^{fpl}* *In applying mathematics. In applied mathematics.*

matrix entry (pl . entrie s), *coefficient d'une matrice*

Matrix norm, *norme matricielle*

Matrix pl matrices [ˈmeɪtrɪks], *matrice^f*

Maximal [ˈmæksɪml], *maximal^{adj}* Maximal element

Maximum [ˈmæksɪməm], pl **maximums or maxima,** *maximum^m*

Maximum principle, *principe du maximum*

Measurable [ˈmeʒərəbl], *mesurable^{adj}*

Measure [ˈmeʒər], *measure^f*

Measure and integration

member [ˈmembər]

Method [ˈmeθəd], *méthode^f*

Methods for Eigenvalues of Symmetric Matrices

Metric [ˈmetrɪk], or distance function, *métrique.*

Metric space, *un espace métrique.*

Minimal [ˈmɪnɪml], *minimal*

minimization [ˌmɪnɪmaɪˈzeɪʃən], *minimisation^f*

Minimization of Convex Functions

Minimum ['mɪnɪməm], pl **minimums** or **minima**, *minimum*^m

modelling, modeling US ['mɒdlɪŋ] *modélisation*^f

modulo, *modulo*

Modulus ['mɒdjʊləs], pl **moduli** ['mɒdjʊ,lɑɪ], *Mathematics, Physics, module*^m

Monotone ['mɒnətəʊn], *Mathématique, monotone*^{adj}

Monotone matrix, *matrice monotone*

Monotonous, [mə'nɒtənəs], *monotone*^{adj}

multi-index *multiindice*

multi-linear form, *forme multilinéaire*

Multilinear, *multilinéaire*^{adj}

multiple ['mʌltɪpl], *Mathematics multiple*^m

multiple ['mʌltɪpl], *Mathematics, multiple*^m

multiple root *racine multiple*

multiplication [ˌmʌltɪplɪ'keɪʃən], *multiplication*^f

multiplicative ['mʌltɪplɪ,keɪtɪv] *Mathematics, multiplicatif*

Multiplicities of an eigenvalue

multiplicity [ˌmʌltɪ'plɪsɪtɪ], *la multiplicité*^f

Multiplied by, Times, *fois, 3 fois 4.*

Multiply ['mʌltɪplaɪ], *multiplier, fois*

N

namely ['neɪmlɪ] *adverb*

Natural ['nætʃrəl], *naturel*, *entier*

Natural numbers : 0,1,2,...

near [nɪəʳ]

Necessary ['nesɪsəri], *nécessaire*

Necessary condition, *condition nécessaire*. a necessary and sufficient condition, *une condition nécessaire et suffisante*

Negative ['negətɪv], *négatif*, *négative*.

Neighbourhood ['neɪbəhʊd], neighborhood US, *voisinage*

Neutral ['nju:trəl] neutral element, *l'élément neutre*

neutrix ['nju:trɪks] **neutrices** noun a neutrix is an additive convex subgroup of \mathbb{R}

Nil [nɪl] noun *zéro*

Non- [nɒn], *non*, *non linéaire*, *nonnegative*, *nonempty*, ...

Noncommutative, *nonnegative*, ...

non-constant, *non constant(e)*^{adj}

non-degenerate, nondegenerate *non dégénéré(e)*

Nonempty set, *un ensemble non vide*.

Nonhomogeneous

Nonlinear [ˌnɒnˈlɪnɪəʳ], *non linéaire*

non-linear, nonlinear, *non linéaire*

Nonlinear Systems and Numerical Optimization

Nonzero vector, *un vecteur non nul*

Norm [nɔ:m], *la norme*

Normal ['nɔ:məl], *normal*

Normed linear space, *espace vectoriel normé*

Normed space [nɔ:məd], *espace normé*

Norms and condition numbers

Notation [nəʊ'teɪʃən], *Mathematics*, *notation*^f

n-th [enθ], the *n*th *le n-ième*

n-th derivative, *dérivée n-ième*

nth prime, The *n*th prime number, *le n-ième nombre premier*.

n-tuple, *n-uplet*

null [nʌl], *nul, nulle*^{adj}

Number ['nʌmbə], *nombre*^m **Number theorist**, **Number Theory**

Numerator ['nju:məreɪtə], *Mathematics*, *numérateur*^m

Numerical [nju:'merɪkəl], *Analyse numérique*

Numerical integration, *intégration numérique*

Numerical Solution of Ordinary Differential Equations



object ['ɒbdʒɪkt]

obvious ['ɒbvɪəs], *évident*

Odd [ɒd], *impair, un entier impair, une fonction impaire*

Odd function, *fonction impaire*

ODE, Ordinary Differential Equations

on the other hand *d'autre part*

Open ['əʊpən], *ouvert*

Operation [ˌɒpə'reɪʃən], *opération*^f

Operator ['ɒpəreɪtə], *opérateur*^m

Optimization [ˌɒptɪmaɪ'zeɪʃən], *optimisation*^f

orbit ['ɔ:bɪt], *orbite*^f

Order ['ɔ:də], *ordre*^m

Order relation, *relation d'ordre*

ordered pair *couple ordonné*

Ordinary ['ɔ:dnrɪ], *ordinaire*

origin ['ɒrɪdʒɪn] *noun* *origine*^f

Orthogonal [ɔ:'θɒgənəl], *orthogonal, polynôme orthogonaux, matrice orthogonale*

Orthonormal basis, *une base orthonormée*

Orthonormal, *orthonormé-e*

Orthonormalization *Orthonormalisation, Orthonormalisation*

Gram-Schmidt orthonormalisation

Otherwise ['ʌðəwaɪz] *adverb*, *autrement*

Otherwise ['ʌðəwaɪz], *sinon*

Over ['əʊvə], *sur*

P

pair [peə], *couple*

Parameter [pə'ræmɪtəʳ], Mathematics, *paramètre*^m

part [pɑ:t], *partie*^f

Partial ['pɑ:ʃəl], *partiel*

partial derivative *dérivée partielle*

Partial Differential Equations

partial sum *somme partielle*

partial sum *somme partielle*

Particular [pə'tɪkjʊləʳ], *particulier, particulière*

Partition [pɑ:tɪʃən], *partition*, *Partition theory is the hardest branch of number theory*

path [pɑ:θ], *chemin*^m

PDE [pi: di: i:], Partial Differential Equations, *EDP*

perfect ['pɜ:fɪkt] *adjective* *parfait* *there is a hard problem with odd perfect numbers*

Plagiarism ['pleɪdʒjəɪzəm] *noun* *plagiat*^m

plane [pleɪn;], Mathematics, *plan*^m

Plus [plʌs], *plus*

PMI Principle of Mathematical Induction

Polar ['pəʊləʳ], *polaire*^{adj}

Polynomial [,pɒlɪ'nəʊmɪəl], *polynôme*^m

Polynomial interpolation, *polynôme et interpolation*

Positivity, *positivité*.

Potential [pəʊ'tenʃəl], *adjective*, Mathematics, *potentiel*.

Power ['paʊə], *puissance*^f

powerful ['paʊəfʊl] *adjective* 108 is a powerful number,
puissant^{adj}

pre... [pri:] *prefix* *pré...*

Previous ['pri:vɪəs], *précédent*, look the previous formula.

Prime [praɪm], *Mathématique*, *prime*, *f* **prime** : *f* *prime*

Prime [praɪm], *premier*

prime number, *un nombre premier*, 2, 3, 5, 7, 11, 13, ...

Primitive ['prɪmɪtɪv], The primitive root of a, *primitif*^m

Principle ['prɪnsəpl], *principe*^m

Probability [ˌprɒbə'bɪlɪtɪ], *probabilité*^f

Problem ['prɒbləm], *problème*^m

process ['prɒsɪs] *processus*^m

Product ['prɒdʌkt], *produit*^m

progression [prə'greʃən] *noun*; in general, *Mathematics*
progression^f **arithmetic progression**, *progression arithmétique*
geometric progression, *progression géométrique*

Proof [pru:f], *démonstration*^f, *preuve*^f

Property ['prɒpətɪ], *propriété*^f

proposition [ˌprɒpə'zɪʃən], *proposition*

prove [pru:v], *prouver*, *démontrer*

Prove that, *prouver que*, *montrer que*, *démontrer que*.

provided that *à condition que*

pseudo- ['sju:dəʊ] prefix *pseudo-*

pure [pjʊə] adjective *pur-e*

purpose ['pɜ:pəs], *but*^m, *objet*^m

Q

Quadratic [kwɒ'drætɪk], *quadratique*^{adj}

Quadratic forms, *formes quadratiques*

Quadrature ['kwɒdrətʃə], *quadrature*^f

Quantity ['kwɒntɪtɪ], *quantité*^f

Quasi- ['kweɪzɪ], *quasi-*, *norms and quasi-norms*

Question ['kwestʃən], *question*^f

Quotient ['kwɒʃənt], *Mathematics*, *quotient*^m

R

R. H.S. [= right hand side] *terme de droite*

Radius ['reɪdɪəs] noun, pl **radiuses**, *rayon*^m

Range [reɪndʒ], the range of f , *L' image = f(E)*, the value of f

Rank [ræŋk], *le rang*

ratio ['reɪʃɪəʊ] noun, *rapport*^m *raison*

rational number ['ræʃənəl], *un nombre rationnel*

Real [riəl], *Mathematics*, *réel*

Real numbers denoted by \mathbb{R} , *On note par \mathbb{R} l'ensemble des nombres réels.*

real-valued function *fonction à valeurs réelle*

reasoning ['ri:znɪŋ], *raisonnement*^m

recall [rɪ'kɔ:l]

Recall that, *rappelons que*

Reciprocal [rɪ'sɪprəkəl], *réci-proque*, *inverse*

Reduction [rɪ'dʌkʃən], *réduction*^f

Reduction of a quadratic form to a sum of squares

Reference ['refrəns], *bibliographie*^f, *référence*^f

reflexive [rɪ'fleksɪv], *Mathématique*, *réflexif*, *-ive*

Regular ['regjʊləʳ], *régulier*

Relation [rɪ'leɪʃən], *relation*

relatively prime *premiers entre eux*

remainder [rɪ'meɪndəʳ], *reste*^m

Remark [rɪ'mɑ:k], *remarque*^f

representation [,reprɪzen'teɪʃən], *représentation*^f

research [rɪ'sɜ:tʃ], *recherche(s)*^{f(pl)}

Residue ['rezɪdju:], *résidu*^m

Resolvable [rɪ'zɒlvəbl] *adjective* *résoluble*

Resolve [rɪ'zɒlv], *résoudre*^v

respectively [rɪ'spektɪvli], resp. *respectivement*

Rest [rest], *le reste*^m

restriction [rɪ'strɪkʃən] *restriction*^f, *limitation*^f

Result [rɪ'zʌlt], *résultat*^m

Riemannian geometry noun *géométrie^f riemannienne*

Riemannian, *riemannien*

Right angle *angle droit*

Ring [rɪŋ], *anneau^m*

Root [ru:t], *Mathematics*, *racine^f* Simple root, *racine simple*, double root *racine double*, triple root *racine triple*, multiple root *racine multiple*, root of multiplicity *m* *racine de multiplicité m*

root of multiplicity m *racine de multiplicité m*

Row [rəʊ], *ligne^f*

row vector *vecteur ligne*

Rule [ru:l], *règle^f*

S

Said [sed], A sequence is said to be Cauchy if, *Une suite est dite de Cauchy, si*

Sample ['sɑ:mpəl], *échantillon^m*

Scalar ['skeɪləɹ] *Mathematics*, *scalaire^{m,adj}*

Scalar product, *Produit scalaire*

Section ['sekʃən], *section^f*, *partie^f*

Self-adjoint [self], *autoadjoint (opérateur)*

Semi- ['semɪ], *semi-*, seminorm, *semi-norme*

Separability, *séparabilité*

Separable ['sepərəəbl], *séparable^{adj}*

Separation [ˌsepə'reɪʃən], *séparation*

Sequence ['si:kwəns], *suite*^f

Series ['siəri:z], *Mathematics*, *série*^f, *suite*^f

seriously ['siəriəslɪ] *adverb* *sérieusement, avec sérieux*

Set [set], *collection*^f, *ensemble*^m

Set of n -th degree polynomials, *L'ensemble des polynômes de degré n .*

Setting ['setɪŋ]

Setting ['setɪŋ], *posons, on pose*

Several ['sevrəl], *plusieurs, several variables, plusieurs variables*

Show that [ʃəʊ], *montrer que*

Sign [saɪn], *le signe*^m

Similar ['sɪmɪlə], *semblable*^{adj}

Similar matrices, *matrices semblables*

similarly ['sɪmɪləli] *adverb, de la même façon, de façon similaire*

Similarly, we have, *de la même façon, on a*

Simplification [ˌsɪmplɪfɪ'keɪʃən], *simplification*^f

Simultaneous [ˌsɪmɪl'teɪnɪəs], *simultané, simultanément*

Simultaneous nonlinear equations

since [saɪns], *comme, puisque*

Since f is linear, comme f est linéaire

Sine [saɪn], *sinus, sine x , sinus x*

situation [ˌsɪtʃʊ'eɪʃən] *noun situation*^f

skew [skju:], *anti-*

skew-symmetric, *anti-symétrique*

Solution [sə'lu:ʃən], *solution*^f

Solution of systems of linear equations

solve [sɒlv], *résoudre*^v

Some [sʌm], some examples, *quelques exemples*

Space [speɪs], *un espace*

Special ['speʃəl], *spécial, particulier*

Special matrices

Spectral ['spektrəl], *le rayon spectral*

Spectral analysis, *Analyse spectrale*

Spectre, **specter**^{US} ['spektər], *spectre*^m

Square [skwɛər], *carré*^m

Square matrix of order n , *matrice carrée d'ordre n* .

Squarefree numbers, *libre de carrés*

Standard ['stændəd], *standard*,

Standard basis, *la base canonique*

Step [step], *étape*, two steps, *deux étapes*

Strictly ['striktli], *d'une manière stricte*

strictly increasing function *fonction strictement croissante*

Strictly less than, *strictement inférieur-e à*

strictly monotone function *fonction strictement monotone*

Strong [strɒŋ], *fort*^{adj}

Strong convergence and weak convergence

Structure ['strʌktʃəʀ], *structure*^f

Study ['stʌdɪ], *étude*^f

Sub [sʌb], *subsequence, subspace,, sous-suite, sous-espace,*

subgroup ['sʌbgru:p], *sous-groupe*^m

subgroup ['sʌbgru:p], *sous-groupe*^m

subject ['sʌbdʒɪkt]

Subsequence, *sous-suite*^f

Subsequent ['sʌbsɪkwənt], *in the subsequent chapters.*

subset ['sʌb,set], *sous-ensemble*^m

Subspace ['sʌb, speɪs], *sous-espace*^m

Subspecies ['sʌb,spi:ʃi:z] pl *sous-espèce*^f

substitution [sʌbstɪ'tju:ʃən], *remplacement*^m, *substitution*^f

subtract [səb'trækt] *verb soustraire*

Successive [sək'sesɪv] *adjective successif*

Successive [sək'sesɪv], *successive itérations.*

Such that, *tel que, tels que, telle que, telles que*

Sufficient [sə'fɪʃənt], *suffisant*^{adj} Sufficient condition

Summation [sʌ'meɪʃən], *addition*^f

Sup [sʌp], *sup, maths, supérieur, the sup of A, le sup de A*

superior [sʊ'pɪəriəʀ]

surface ['sɜ:fɪs] *surface*^f

Surjective [sɜː'dʒektɪv], *surjectif*^{adj}

Symmetric [sɪ'metrɪk], *Mathematics*, *symétrique*

Symmetric positive definite matrices

Symmetrically [sɪ'metrɪkəlɪ] *adverb* *symétriquement*, *avec symétrie*

Symmetry ['sɪmɪtri] *noun* *symétrie*^f

System ['sɪstəm], *système*^m

T

Table ['teɪbl], *tableau*^m, *liste*^f

tangent ['tændʒənt] *noun*, *Mathematics*, *tangente*^f

TD [tɪr'diː], *abréviation de travaux dirigés* (Université)

Technique [tek'nɪk], *technique*^f

tend [tend]

The dimension of a vector space

The intersection of S and T , the union of S and T.

the Laplace operator *opérateur de Laplace*

The set ofsuch that, {*The set of ... such that...*},
L'ensemble de ...tel que

theme [θiːm], *thème*^m, *sujet*^m

Theorem ['θɪərəm], *théorème*^m

theoretician [θɪərə'tɪʃən] **theorist** ['θɪərɪst] *noun*

Theory ['θɪəri], *théorie*^f

Therefore [ˈðeəfɔːr], *donc, par conséquent*

This means, *c'est-à-dire*

PhD [ˌpiːɪtʃˈdiː] **Univ** abbreviation of *Doctor of Philosophy* = qualification *doctorat*^m to have a PhD in ...: *avoir un doctorat de ...*

throughout [θrʊˈaʊt] **preposition**, *partout dans*

Times [taɪmz], *multiplier, fois, 3 times 4, 3 fois 4*

To present, to show, to prove, ...

Topologic [ˌtɒpəˈlɒdʒɪk], **topological** [ˌtɒpəˈlɒdʒɪkəl], *topologique*^{adj}

Topological space, *espace topologique*^{adj}

Topology [təˈpɒlədʒɪ], *topologie*^f

total [ˈtəʊtl]

Trace [treɪs], *la trace*^f

Trace, the trace of a matrix, *la trace d'une matrice*

Transcendental [ˌtrænsenˈdɛntl], *transcendant*

Transcendental number, *un nombre transcendant*

transformation [ˌtrænsfəˈmeɪʃən] **noun** Mathematics, Physics, Linguistics **transformation**^f

Transitive [ˈtrænzɪtɪv] , *transitif*^{adj}

Transpose [trænsˈpəʊz], *transposer*

Transpose, A transpose, *A transposée*

Triangle [ˈtraɪæŋɡl], *triangle*^m

Triangle inequality, *inégalité triangulaire*

Triangular [traɪ'æŋgjʊləɹ], *triangulaire*^{adj}

Tridiagonal matrices

tridimensional [,traɪdɪ'menʃənl], *tridimensionnel, à trois dimensions*

Trigonometric formulae, *formules trigonométriques*

Trigonometric, [,trɪgənə'metrɪk], **trigonometrical**
[,trɪgənə'metrɪkəl], *trigonométrique, série trigonométrique*

trilinear form, *forme trilinéaire*

triple ['trɪpl], *triplet*

Trivial ['trɪvɪəl], *trivial, -e, mpl -iaux*

Twice [twɑɪs], *deux fois*

twice differentiable function *fonction deux fois dérivable*
 n -times continuously differentiable function *fonction n fois
continument dérivable*

twin [twɪn], **twin primes**, *nombres premiers jumeaux*

U

Unbounded [ʌn'baʊndɪd], *illimité, non borné*

Unbounded operator, *opérateur non borné*

Uncountable ['ʌn'kaʊntəbl], *non dénombrable, the set of real
numbers is not uncountable.*

understand [ʌndə'stænd] **understood**

Unicity, *unicité*^f

Uniform ['juːnɪfɔːm], *uniforme*

Uniformly ['juːnɪfɔːmlɪ], *uniformément*, a map uniformly continuous,
application uniformément continue.

Union ['juːnjən], *union*

Unique [juː'niːk], *unique*^{adj}

uniquely [juː'niːklɪ] *adverb*

Uniqueness [juː'niːknɪs], *unicité*^f

Unit ['juːnɪt], *unité*^f

Unitary ['juːnɪtəri], *matrice unitaire, groupe unitaire, application unitaire*

Unknown ['ʌn'nəʊn], *inconnu*^{adj}

Unlimited [ʌn'lɪmɪtɪd], *illimité*^{adj}

unresolved ['ʌnrɪ'zɒlvd] = *unsolved, problem, non résolu*

Upper ['ʌpə], *upper bound, la borne supérieure*

Upper triangular matrix, *matrice triangulaire supérieure*

Using integration by parts gives,

Using the last equation gives

Using theorem 1.2, *En utilisant le théorème 1.2*,

usual ['juːʒʊəl]



Value, values ['væljuː], *valeur*^f

Variable ['vɛəriəbl], *variable*

Variation [ˌvɛəri'eɪʃən], *variation*^f

variety [və'raɪəti] *noun variété*^f

Various ['vɛəriəs] *différent*

Vect ['vekt] *Vect*

Vector ['vektər], *Mathematics*, *vecteur*^m

vector space of dimension n , *espace vectoriel de dimension n*

Vector space, *un espace vectoriel ou un espace vectoriel normé*

vector subspace, *sous-espace vectoriel*

verification [,verɪfɪ'keɪʃən] = *check*, *vérification*^f,

viewpoint ['vjʊ:pɔɪnt], *point*^m *de vue*

Volume ['vɒljʊ:m], *noun*, *volume*^m

W, Z, X

Wave [weɪv], *wave equation*, *équation des ondes*

We denote by, *on note par*

We distinguish two cases, *On distingue deux cas*

We have, we've, *on a, nous avons*

We obtain, *on trouve*

We put, Put, Setting, *posons, on pose*

We see that, *on voit que*

weak [wi:k], *faible*, *weak convergence*

Weak topology, *la topologie faible*

whatever [wɒt'evər]

whence [wens] *conjunction* *d'où*

Whence, *hence, therefore, and hence* [wens], *d'où*

where, where p is an odd prime, *où*

whereas [wɛər'æz] **conjunction** = while *alors que, tandis que*

whereby [wɛə'baɪ] **pronoun** *par quoi, par lequel* (or *laquelle* etc), *au moyen duquel* (or *de laquelle* etc)

whether ['weðə], *si* **which** [wɪtʃ] **whichever** [wɪtʃ'evə]

while [waɪl] = during the time that *pendant que*

whole [həʊl] **adjective** = entire *tout, entier*

whose [huːz] **possessive pronoun** *à qui*

with respect to [= w.r.t.], *par rapport à*

wlog = without loss of generality

Work [wɜ:k], *travail*, in this work we prove that ..., *dans ce travail montrons que*

X, x [eks], x to the power n , *X to the n*, *X à la puissance n*.

zero ['zɪərəʊ], pl **zeros** or **zeroes** **noun** *zéro*^m

zeta *zeta* zeta function

Part 4, Nonordinary English words

In the following section we ask whether we can read correctly the following mathematical words or not, without seeing the ordinary English words.

4.1. Nonordinary mathematical words

Problem 1. Re-write the following mathematical words in ordinary English.

A [ə 'set] [ə'blʌv] ['æbsəlu:t] ['ædɪŋ] [ə'dɪʃən] [əd'mɪt] ['ældʒɪbrə] [ældʒɪ'breɪɪk] ['ælgə,rɪðəm] ['ɒltənɛɪtɪŋ] [ə'næləgəs] [ə'næləsɪs] [ə'nælɪsɪz] ['aɪnsə] [æplɪ'keɪʃən] [ə'prəʊtʃ] ['a:bɪtrəri] [ə,prɒksɪ'meɪʃən] [a:k] ['a:ɡjəmənt] ['a:ɪkɪl] [ə'rɪθmətɪk] [ə'saɪn] [ə'səʊʃɪətɪv] [ə'sju:m] ['æksɪəm].

B [bɔ:l] ['beɪsɪk] ['beɪsɪs] [bɪ'ləŋ] [baɪ'dʒektɪv] [sɜ:'dʒektɪv] [ɪn'dʒektɪv] ['baɪnəri] [baɪ'nəʊmiəl] [baɪ'sekʃən] [bæʊnd] ['bæʊndəri] [brɑ:ntʃ] [bʌt].

C ['kælkjʊləs] [kə'nɒnɪkəl] [kɑ:'ti:ziən] ['tʃeɪndʒɪŋ] ['tʃæptə] [,kæɪktə'rɪstɪk] [,kæɪktərəɪ'zeɪʃən], ['sɜ:kl] [sə'kʌmfərəns] [kla:s] [kləʊzd] [kəʊ] [,kəʊ'fɪʃənt] ['kəʊ,fæktə] [kə'lekʃən][kə'ləm] [laɪn] [,kɒmbɪ'neɪʃən] ['kɒment] [kə'mju:tətɪv] [kəm'pækt] [kəm'pærɪsn] ['kɒmplɪmənt] ['kɒmpleks] ['nʌmbə'z] [kəm'pəʊnənt] ['kɒmpəzɪt] ['kɒnsept] [kən'klu:d] [kən'klu:zən] [kən'dɪʃən] ['kɒndʒʊgeɪt] [kən'dʒektʃə] ['kɒnsɪkwəns] [kən'sɪdə] ['kɒnstənt] [kən'teɪn] [kən'tɪnjʊəs] [kən'trækʃən] [,kɒntrə'dɪkʃən] ['kɒntent] ['kɒntʊə] [kən'vɜ:dʒ] [kən'vɜ:dʒəns] [kə'nekt] ['kɒn'veks] [,kɒnvə'lu:ʃən] [kə'rɒləri] ['kəʊsaɪn] [saɪn] ['kəʊntəbl][kəʊ'veəriəns] [kraɪ'tɪəriən][kraɪ'tɪəriə] [krɒs] ['saɪklɪkəl] ['kju:bɪk] [kɜ:v].

D [,di:kɒmpə'zɪʃən] [di:'kri:siŋ] [dɪ'faɪn] ['defɪnɪt] [,defɪ'nɪʃən][dɪ'gri:] ['deltə] ['demənstreɪbl] [,demən'streɪʃən] [dɪ'nɒmɪneɪtə] [dɪ'nəʊt] ['densɪtɪ] [dɪ'pendəns] [,ɪndɪ'pendəns] [,derɪ'veɪʃən] [dɪ'rɪvətɪv] [dɪ'tɜ:mɪnd] ['ʌndɪ'tɜ:mɪnd] [dɪ'tɜ:mɪnənt] [dɪ'veləpmənt] [daɪ'ægənəl] ['daɪəgræm] ['dɪfrəns] [,dɪfə'renʃəl] [,dɪfərənʃɪ'eɪʃən] ['dɪfɪkəlt], ['dɪdʒɪt] [daɪ'rekt] [dɪ'rekt] [dɪs'kri:t] [,dɪskən'tɪnjʊəs] [,dɪskɒntɪ'nju:ɪtɪ] [dɪs'kʌʃən] [dɪs'dʒɔɪnt] ['dɪstəns] [dɪs'tɪŋgwɪʃ] [dɪstrɪ'bju:ʃən] [daɪ'vɜ:dʒ] [daɪ'vɜ:dʒəns] [dɪ'vaɪdɪd] [dɪ'vɪzəbl] [dɪ'vɪzən] [dɪ'vaɪzə] [dəʊ'meɪn] ['dʌbl] [dɒt] ['dʒʊəl] [dʒʊ'æli:tɪ].

E ['i:zi] [eɪzən pɛə] [eɪzən 'vælju:] [eɪzən 'vektə] [eɪzən ['fʌŋkʃənz] [eɪzən speɪs] ['elɪmənt] [ɪ,lɪmɪ'neɪʃən] [ɪ'lɪps] [ɪ'lɪptɪk] [ɪ'lɪptɪkəl] ['emptɪ] [,endəʊ'mə:fɪzəm] ['envələʊp] ['i:kwəl] [ɪ'kwɒli:tɪ] [ɪ'kwɛɪzən] [,ɪkwɪ'lɪbrɪəm] [ɪ'kwɪvələns]['erə] [,estɪ'meɪʃən] [ju:'klɪdɪən] [ɪ'væljuəɪt] [ɪ'vən] [ɒd] [ɪ'vent] ['evidənt] ['ɒbvɪəs] [ɪg'zækt] [ɪg'zɑ:mpl] ['eksəsaɪz] [ɪg'zɪstəns] [ɪk'spænʃən] [ɪk'sperɪmənt] [ɪk'spres] [ɪk'stenʃən] [ɪk'stremɪtɪ] [ɪk'stɪəriə].

F ['fæktə] ['fæmɪli] ['fæktə,raɪz] ['fɪnɪʃ][fɜ:st] ['fɜ:stli][fɪkst] ['fɒləʊɪŋ] [fɔ:r] [fɔ:m] ['fɔ:mjʊlə] ['frækʃən]['frækʃənl] [fri:] ['fʌŋkʃən] ['fʌŋkʃənl] [,fʌndə'mentl]

G ['gæmə] [gəʊs] ['dʒenərəl] [,dʒenərələɪ'zeɪʃən] [dʒɪ'ɒmɪtɪ] ['gləʊbl] ['greɪdɪənt] [grɑ:f] [gru:p] [rɪŋ] ['bɒdi] [fi:ld].

H [ˈaʊər] [hɑːˈmɒnɪk] [hɪt][hɪnt] [ˌhəʊməˈdʒiːniəs] [həˈmɒdʒɪnəs]
[həʊmiəˈmɔːfɪzəm] [ˌhɒməˈmɔːfɪzəm] [hɑɪˈpɜːbələ] [ˌhaɪpəˈbɒlɪkəl] [ˌhaɪpəˈbɒlɪkəl]
[hɑɪˈpɜːbəlɔɪd] [ˌhaɪˈpɒθɪsɪs] [hɑɪˈpɒθɪsɪz] [aɪˈdiə] [aɪˌdentɪfɪˈkeɪʃən] [aɪˈdentɪtɪ]
[ɪˈmiːdiət] [ˌɪmplɪˈkeɪʃən] [ɪmˈplɪsɪt] [ɪmˈplɑɪ] [ɪmˈprɒpər] [ɪn] [əˈkɔːdəns]
[ɪnˈkluːd] [ɪnˈkriːsɪŋ] [ɪnˈdefɪnɪt] [ˌɪndɪˈpendəns] [ɪnˈdʌkʃən] [ɪnˈdʌktɪv]
[ˌɪnɪˈkwɒlɪtɪ] [ˌɪnfɪnɪˈtesɪmə] [ɪˈnɪʃəl] [ɪnˈdʒekʃən] [ˌɪnˈhəʊməˈdʒiːniəs] [ˈɪnər]
[ˈɪntɪdʒər] [ˈɪntɪgrəl] [ˌɪntɪˈgreɪʃən] [ˈɪtərətɪv] [ɪnˈtɪəriər] [ɪnˌtɜːprɪˈteɪʃən]
[ɪnˌtɜːpəˈleɪʃən] [ɪntəˈsekʃən] [ˈɪntəvəl] [ɪnˈvɛəriənt] [ˈɪnvɜːs] [ɪˈræʃən]
[ˌaɪsəʊˈmɔːfɪzəm]

K [ˈkɜː], [ˈkɜːnl].

L [lɑːst] [lɔː] [ləˈgreɪnd] [ləˈgreɪndʒɪən] [ləˈplæs] [liːst] [ˈlemə] [leŋ(k)θ] [let] [ˈlɪmɪt]
[ˈlɪnɪər] [ˈləʊkəl] [lɒg] [ˈlɒgəriθəm] [ˈləʊər baʊnd].

M [ˈmæp] [ˈmæpɪŋ] [mæs] [mæθ] [ˌmæθəˈmætɪkəl] [ˌmæθəˈmætɪkəlɪ]
[ˌmæθəməˈtɪʃən] [mæθs] [ˌmæθəˈmætɪks] [ˈmeɪtrɪks] [ˈmæksɪml], [ˈmæksɪmə]
[ˈmæksɪmə] [ˈmeɪzər] [ˈmeθəd] [ˈmetrɪk] [ˈmɪnɪmə] [ˈmɪnɪmə] [ˈmɒdʒləs]
[ˈmɒnətəʊn] [ˈmɔːfɪzəm] [ˌməʊtɪˈveɪʃən] [ˈmʌltɪ] [ˌmʌltɪplɪˈkeɪʃən] [ˈmʌltɪplɑɪd]
[ˌmʌltɪˈplɪsɪtɪ].

N [ˈnegətɪv] [ˈpɒzɪtɪv] [ˈneɪbəhʊd] [njuː] [nɒn] [ˌnɒnˈlɪnɪər] [nɒnkəˈmjuːtətɪv]
[nɔːm] [ˈnɔːmə] [nəʊˈteɪʃən] [ˈnjuːməreɪtər] [ˈnʌmbər] [njuːˈmerɪkəl]
[njuːˈmerɪkəlɪ].

O [ˈɒbdʒɪkt] [ˈɒbvɪəs] [ˈɒfən, ˈɒftən][ɒn] [ˈəʊpən] [ˌɒpəˈreɪʃən] [ˈɒpəreɪtər]
[ˈɒpəzɪt] [ˈɒrɪdʒɪn] [ɔːˈθɒgənɪ] [ɔːθəˈnɔːmə]

P [pəˈræmɪtər] [ˌpærəˈmetrɪk] [ˌpærəˈbɒlɪk] [pəˈrenθɪsɪs] [pəˈrenθɪsɪz] [pəˈtɪkjʊlə]
[pəˈtɪʃən] [pəˈsent] [ˈpɪəriəd] [ˌpɜːmɪʃˈteɪʃən] [ˌkɒmbɪˈneɪʃən] [ˌpɪəriˈɒdɪk] [piːs]
[fɪˈnɒmɪnə] [plʌs] [ˈmaɪnəs] [ˈpwaːsən] [ˈpəʊlə] [ˌpɒlɪˈnəʊmɪəl] [priː] [praɪm]
[ˈprɪmɪtɪv] [ˈprɪnsɪpəl] [ˌprɒbəˈbɪlɪtɪ] [ˈprɒbləm] [ˈprəʊses] [ˈprɒdʌkt] [prəˈgreʃən]
[prəˈdʒekʃən] [pruːf] [ˈprɒpətɪ] [prəˈvaɪdɪd] [pɜːsər] [paɪˈθæɡəˈrɪən].

Q [kwɒˈdrætɪk] [ˈkwɒntɪtɪ] [ˈkweɪzɪ] [ˈkwestʃən] [ˈkwɒʃənt].

R [ˈreɪdɪəs] [ˈɛəriə] [səˈkʌmfərəns] [ˈrændəm] [ræŋk] [ˈræʃənɪ] [rɪəl] [rɪˈkɔːl]
[rɪˈsɪprəkəl] [ˈrek,tæŋɡl] [rɪˈkʌrəns] [ɪnˈdʌkʃən] [rɪˈfleksɪv] [sɪˈmetrɪk]

['trænzɪtɪv] [rɪ'fleksɪv] ['regjʊləʳ] [rɪ'leɪʃən] ['relətɪv] [,reprɪzen'teɪʃən] ['rezɪdju:]
[,rezə'lu:ʃən] [rɪ'zɒlv] [sɒlv] [rest] [rɪ'zʌlt] [rɪ'mæniən] [rɪŋ] [ru:t] [rəʊ'teɪʃən] [ru:l].

S ['sa:mpəl] ['skeɪləʳ] [ʃəʊ] [sɜ:tʃ] ['sɪ:kənt] ['sekənd] ['semɪ] ['sepərəbəl] ['sɪ:kwəns]
['sɪəri:z] ['sɪ:kwəns] [set] [saɪn] [sɪg'nɪfɪkənt] ['sɪmɪləʳ] [ˌsɪməl'teɪniəs] ['sɪŋɡl]
['sɪŋɡjʊləʳ] [sləʊp] [sə'lu:ʃən] [sɒlv] [ɪ'nɪʃəl], [speɪs] ['spektrəl] [skweəʳ] [skweəʳ]
[stə'bɪlɪtɪ] ['steɪʃənəri] [stə'tɪstɪk] [stə'kæstɪk] ['strʌktʃəʳ] ['stʌdɪ] ['sʌbgru:p]
['sʌb,set] ['sʌb,speɪs] ['sʌb,spi:ʃi:z] ['sʌbstɪtju:t] [ˌsʌbstɪ'tju:ʃən], [sʌm] [sʌ'meɪʃən]
['sʌməraɪz] [sʌ'meɪʃən] [sʌp] [sɜ:d] | [ɪ'ræʃənəl] ['sɜ:fɪs] [sɪ'metrɪk], [sɪ'metrɪkəl]
['sɪmɪtrɪ] ['sɪstəm].

T ['teɪbl] ['tændʒənt] [tek'ni:k] [taɪmz] [ðen] ['θɪərəm] [θɪə'retɪkəl] [θɪə'retɪkəlɪ]
['θɪəri] [æd] [ə'laʊ] ['pɜ:mɪt] ['ɪ:kwəl] ['lɒdʒɪk] [tə'pɒlədʒɪ] [treɪs] [træns'pəʊz]
[,trænsen'dentl] [trə'pi:ziəm] [trə'pi:ziə] ['træpɪzɔɪd] ['traɪæŋɡl] [,traɪdaɪ'ægən]
[,traɪdɪ'menʃən] [,trɪgənə'metrɪkəl] [,trɪgənə'metrɪkəl] ['trɪpl] [twəɪs].

U, V, W [ʌn'baʊndɪd] ['ʌn'kaʊntəbl] ['ʌndɪ'tɜ:mɪnd] ['ju:nɪfɔ:m] ['ju:njən]
['ʌn'mɪkst] [ju:'nɪ:knɪs] ['ju:nɪt] ['ʌn'nəʊn] ['ʌpəʳ bəʊnd] [ˌju:nɪ'vɜ:səl] ['vælju:]
['vælju:d] ['veəriəbl] ['veəriəns] [ˌveəri'eɪʃən] ['vaɪə] ['vektəʳ]
[weɪv] [hi:t] [weɪ] ['mænəʳ] [wɪ:k] ['wɪ:kli] [wen] [wɪð, wɪθ] [wɜ:k] ['zɪərəʊ].

4.2. Problems with phonetic symbols

Problem 1. Read correctly the following words.

Example 02. The following are used in Mathematics

- **generalization**
- **Optimization**
- **Comparison**
- **Approach**
- **Solution**
- **differential**
- PDE
- ODE
- **Operator**
- **rational number**
- **Spectral**
- **Exponential**
- **Polynomial**

Problem 2. Read correctly the following words.

[,dʒenərəlaɪ'zeɪʃən]

[,ɒptɪmaɪ'zeɪʃən]

[kəm'pærɪsn]

[ə'prəʊtʃ]

[sə'lu:ʃən]

[,dɪfə'renʃəl]

PDE

ODE

['ɒpərəɪtət̩r]

['ræʃən]

['spektrəl]

[,ekspəʊ'nenʃəl]

[,pɒlɪ'nəʊmɪəl]

Problem 3. Read correctly the following words.

Djamel ['dʒæmel]

child [tʃaɪld]

wild [waɪld]

milk [mɪlk]

level ['levl]

pupil ['pjʊ:pəl]

full [fʊl]

real [riəl]

small [smɔ:l]

call [kɔ:l]

lunch [lʌntʃ]

several ['sevrəl]

natural ['nætʃrəl]

general ['dʒenərəl]

○ ['meɪtrɪks]

○ ['mæksɪml]

○ ['mæksɪməm],

○ ['meʒərəbl]

○ ['meʒər]

○ ['meθəd]

- ✓ **Matrix** ['meɪtrɪks]
- ✓ **Maximal** ['mæksɪml]
- ✓ **Maximum** ['mæksɪmə],
- ✓ **Measurable** ['meʒərəbl]
- ✓ **Measure** ['meʒə]
- ✓ **Method** ['meθəd]

- ✓ **Algeria** [æ'lʒɪəriə]
- ✓ **Belgium** ['belʒəm]
- ✓ **wouldn't** ['wʊdnt]
- ✓ **April** ['eɪprəl]
- ✓ **day** [deɪ]
- ✓ **government** ['gʌvənmənt]
- ✓ **substitution**
[,sʌbstɪ'tjuːʃən]
- ✓ **suggest** [sə'dʒest]

- [æ'lʒɪəriə]
- ['belʒəm]
- ['wʊdnt]
- ['eɪprəl]
- [deɪ]
- ['gʌvənmənt]
- [,sʌbstɪ'tjuːʃən]
- [sə'dʒest]

- ['mɪrə]
- [laɪdʒ]
- [lɑːf]
- [ɪ'nʌf]
- [ɪm'pɔːtəns]
- [,ɪnfə'meɪʃən]

- ['mɪrə]
- [laɪdʒ]
- [lɑːf]
- [ɪ'nʌf]
- [ɪm'pɔːtəns]
- [,ɪnfə'meɪʃən]

- ✓ **mirror** ['mɪrə]
- ✓ **large** [laɪdʒ]
- ✓ **laugh** [lɑːf]
- ✓ **enough** [ɪ'nʌf]
- ✓ **importance** [ɪm'pɔːtəns]
- ✓ **information** [,ɪnfə'meɪʃən]

- ['hɪstəri]
- ['peɪʃənt]
- ['pɔɪznəs]
- [dɪ'zɑːstə]
- ['lɔɪjə]
- ['kwɛstʃən]
- [,kwɛstʃə'nɛə]

- ✓ **history** ['hɪstəri]
- ✓ **patient** ['peɪʃənt]
- ✓ **poisonous** ['pɔɪznəs]
- ✓ **disaster** [dɪ'zɑːstə]
- ✓ **lawyer** ['lɔɪjə]
- ✓ **question** ['kwɛstʃən]
- questionnaire** [,kwɛstʃə'nɛə]

- [rɪəl]
- [rʊm]
- [praɪs]
- ['pɪktʃəʳ]
- ['fjuːtʃəʳ]
- [traɪ]
- ['neɪtʃəʳ]
- ['nætʃrəl]
- ['lɪtərəɪtʃəʳ]

- **real** [rɪəl]
- **room** [rʊm]
- **price** [praɪs]
- **picture** ['pɪktʃəʳ]
- **future** ['fjuːtʃəʳ]
- **try** [traɪ]
- **nature** ['neɪtʃəʳ]
- **natural** ['nætʃrəl]
- **literature** ['lɪtərəɪtʃəʳ]

- ['steɪdɪəm]
- ['træfɪk]
- [ɪn'tenʃən]
- ['speʃəlaɪz]
- ['speʃəl]
- [ˌʌndə'laɪn]
- [ˌɪntrə'djuːs]
- [ˌɪntrə'dʌkʃən]

- ✓ **stadium** ['steɪdɪəm]
- ✓ **traffic** ['træfɪk]
- ✓ **intention** [ɪn'tenʃən]
- ✓ **specialize** ['speʃəlaɪz]
- ✓ **special** ['speʃəl]
- ✓ **underline** [ˌʌndə'laɪn]
- ✓ **introduce** [ˌɪntrə'djuːs]
- ✓ **introduction** [ˌɪntrə'dʌkʃən]

- [prə'vɪʒən]
- ['vɪʒən]
- ['levl]
- [ɪl]
- ['leɪtəʳ]
- ['pɑːtɪ]
- ['laɪbrəri]

- **provision** [prə'vɪʒən]
- **Vision** ['vɪʒən]
- **level** ['levl]
- **ill** [ɪl]
- **later** ['leɪtəʳ]
- **party** ['pɑːtɪ]
- **library** ['laɪbrəri]

- ['mjuːzɪk]
- [mjuː'zɪʃən]
- ['feɪməs]
- [pjʊəʳ]
- ['lɪtl]
- ['lɪsn]
- [wɜːd]
- [wɜːld]

- ✓ **music** ['mjuːzɪk]
- ✓ **musician** [mjuː'zɪʃən]
- ✓ **famous** ['feɪməs]
- ✓ **pure** [pjʊəʳ]
- ✓ **little** ['lɪtl]
- ✓ **listen** ['lɪsn]
- ✓ **word** [wɜːd]
- ✓ **world** [wɜːld]

- ['wedɪŋ]
- ['mɔ:nɪŋ]
- ['dʒænjʊərɪ]
- [beɪʒ]
- [peɪdʒ]
- [wɪl]

- [ɪlek'trɪsətɪ]
- ['dʒʌstɪs]
- [ˌdʒʌstɪfɪ'keɪʃən]
- [æt]
- [kaɪnd]
- ['kaɪndnɪs]
- [ˌpɒlɪ'nəʊmɪəl]

- [prə'dju:s]
- ['pɪknɪk]
- [mɪlk]
- ['sevrəl]
- [pɑ:k]
- [kɑːr]
- ['mi:tɪŋ]
- ['mɪnɪstrɪ]
- ['mɪnɪməm]

- ['peɪpəːr]
- [pə'tɪkjʊləːr]
- [pə'tɪkjʊlələɪ]
- ['fræŋklɪ]
- ['prɒpətɪ]
- [ki:]
- [ðeɪ]

- ✓ **wedding** ['wedɪŋ]
- ✓ **morning** ['mɔ:nɪŋ]
- ✓ **January** ['dʒænjʊərɪ]
- ✓ **beige** [beɪʒ]
- ✓ **page** [peɪdʒ]
- ✓ **will** [wɪl]

- ✓ **would** [wʊd]
- ✓ **should** [ʃʊd]
- ✓ **bone** [bəʊn]
- ✓ **potato** [pə'teɪtəʊ]
- ✓ **girl** [gɜ:l]
- ✓ **first** [fɜ:st]
- ✓ **near** [nɪəːr]
- ✓ **here** [hɪəːr]

- ✓ **produce** [prə'dju:s]
- ✓ **picnic** ['pɪknɪk]
- ✓ **milk** [mɪlk]
- ✓ **several** ['sevrəl]
- ✓ **park** [pɑ:k]
- ✓ **car** [kɑːr]
- ✓ **meeting** ['mi:tɪŋ]
- ✓ **ministry** ['mɪnɪstrɪ]
- ✓ **minimum** ['mɪnɪməm]

- ✓ **paper** ['peɪpəːr]
- ✓ **particular** [pə'tɪkjʊləːr]
- ✓ **particularly** [pə'tɪkjʊlələɪ]
- ✓ **frankly** ['fræŋklɪ]
- ✓ **property** ['prɒpətɪ]
- ✓ **key** [ki:]
- ✓ **they** [ðeɪ]

- **patient** ['peɪfənt]
- **poisonous** ['pɔɪznəs]
- **disaster** [dɪ'zɑːstə]
- **lawyer** ['lɔːjə]
- **question** ['kwestʃən]
- **questionnaire** [ˌkwestʃənɛə]

noun

- [wʊd]
 - [ʃʊd]
 - [bʌʊn]
 - [pə'teɪtəʊ]
 - [gɜːl]
 - [fɜːst]
 - [nɪə]
 - [hɪə]
- ['prɒbləm]
 - ['speɪʃəs]
 - [ɪn'tɪəriə]
 - [ɪk'stɪəriə]
 - [ˌkɒnsəl'teɪʃən]
 - ['kʊdnt]
- [mə'tɪəriəl]
 - [ˌmæθə'mætɪks]
 - [ˌmæθəmə'tɪʃən]
 - ['sɪstəm]
 - ['meʒə]
- ['lesn]
 - ['seʃən]
 - ['mæksɪml]
 - [ˌmæksɪmaɪ'zeɪʃən]
 - [mɑːtʃ]
 - ['mesɪdʒ]
- ✓ **problem** ['prɒbləm]
 - ✓ **spacious** ['speɪʃəs]
 - ✓ **interior** [ɪn'tɪəriə]
 - ✓ **exterior** [ɪk'stɪəriə]
 - ✓ **consultation** [ˌkɒnsəl'teɪʃən]
 - ✓ **couldn't** ['kʊdnt]
- ✓ **material** [mə'tɪəriəl]
 - ✓ **mathematics** [ˌmæθə'mætɪks]
 - ✓ **mathematician** [ˌmæθəmə'tɪʃən]
 - ✓ **system** ['sɪstəm]
 - ✓ **measure** ['meʒə]
- ✓ **lesson** ['lesn]
 - ✓ **session** ['seʃən]
 - ✓ **maximal** ['mæksɪml]
 - ✓ **maximization** [ˌmæksɪmaɪ'zeɪʃən]
 - ✓ **march** [mɑːtʃ]
 - ✓ **message** ['mesɪdʒ]
- ✓ **knowledge** ['nɒlɪdʒ]
 - ✓ **lady** ['leɪdɪ]
 - ✓ **comb** [kəʊm]
 - ✓ **language** ['læŋɡwɪdʒ]
 - ✓ **sufficient** [sə'fɪʃənt]
 - ✓ **force** [fɔːs]
 - ✓ **law** [lɔː]
 - ✓ **lazy** ['leɪzɪ]

- ['nɒlɪdʒ]
- ['leɪdɪ]
- [kəʊm]
- ['læŋgwɪdʒ]
- [sə'fɪʃənt]
- [fɔːs]
- [lɔː]
- ['leɪzɪ]
- ✓ **electricity** [ɪlek'trɪsətɪ]
- ✓ **justice** ['dʒʌstɪs]
- ✓ **justification** [ˌdʒʌstɪfɪ'keɪʃən]
- ✓ **at** [æt]
- ✓ **kind** [kaɪnd]
- ✓ **kindness** ['kaɪndnɪs]
- ✓ **polynomial** [ˌpɒlɪ'nəʊmɪəl]

1. **I**nternational [ˌɪntə'næʃnəl]
2. **i**nternationalist [ˌɪntə'næʃnəlɪst]
3. **i**nternationalization [ˌɪntə'næʃnəlaɪ'zeɪʃən]
4. **U**nemployment ['ʌnɪm'plɔɪmənt]
5. **T**ry [traɪ]
6. **f**uture ['fjuːtʃə]
7. **s**atisfy ['sætɪsfaɪ],
8. **t**echnique [tek'nɪk]
9. **g**eneralization [ˌdʒenərəlaɪ'zeɪʃən]
10. **t**oward [tə'wɔːd]
11. **c**lothes [kləʊðz],
12. **s**upporter [sə'pɔːtə]
13. **o**pponent [ə'pəʊnənt],
14. **s**ummarize ['sʌməraɪz]
15. **p**rocedure [prə'sɪdʒə]

1. [ˌɪntə'næʃnəl]
2. [ˌɪntə'næʃnəlɪst]
3. [ˌɪntə'næʃnəlaɪ'zeɪʃən]
4. ['ʌnɪm'plɔɪmənt]
5. [traɪ]
6. ['fjuːtʃə]
7. ['sætɪsfaɪ]
8. [tek'nɪk]
9. [ˌdʒenərəlaɪ'zeɪʃən]
10. [tə'wɔːd]
11. [kləʊðz]
12. [sə'pɔːtə]
13. [ə'pəʊnənt]
14. ['sʌməraɪz]
15. [prə'sɪdʒə]

Problem 4. Read correctly the following words.

pure [pjʊə^r]

cocksure ['kɒkʃʊə^r]

[dɪ'trækʃən]

[spæn]

[məʊl]

[ɪ'tɜːnɪtɪ]

['kɒnfərəns]

['fʌstɪ]

[fjuː'tɪlɪtɪ]

['fjuːtʃəɪzəm]

[dɪ'vaɪs]

['jʊərəʊ]

['feɪməs]

[fə,nætɪsəɪ'zeɪʃən]

[kə'mens]

[kə'mensəlɪzəm]

[ɪn'dʌstrɪəs]

['laɪfə^r]

[ɪg'zɔːlt]

[ɪg'zæmɪnə^r]

['dæm'fuːl]

['klemənsɪ]

[bəʊl]

['bəʊlɪŋ]

[kɒk]

['kɒkəʳ]

['kɒkɪnɪs]

decumulation [ˌdɪkjʊːmjʊ'sleɪʃən]

electrotechnological [ɪˌlektərəʊˌteknə'lədʒɪkəl] *adjective*

electrothermal [ɪˌlektərəʊ'thɜːməl] **electrothermic**

[ɪˌlektərəʊ'thɜːmɪk] *adjective*

electrovalence [ɪˌlektərəʊˌveɪləns], **electrovalency**

[ɪˌlektərəʊˌveɪlənsɪ] *noun*

communize ['kɒmjʊːnaɪz] *transitive verb*

beleaguered [bi'liːgəd] *adjective*

believe [bi'liːv]

Belisha beacon [bi'liːʃə'biːkən]

caryopsis [ˌkæri'ɒpsɪs] *noun* **caryopses** [ˌkæri'ɒpsɪz], , or

caryopsides [ˌkæri'ɒpsɪˌdiːz]

cybersquatting ['saɪbəskwɒtɪŋ]

charmless ['tʃɑːmlɪs] *adjective*

characteristically [ˌkærɪktə'rɪstɪkəlɪ] *adverb*

charbroiled ['tʃɑːˌbrɔɪld] *adjective*

diabolism [daɪ'æbəlɪzəm] *noun*

endocardium [endəʊ'kɑːdɪəm], pl **endocardia**

indiscriminately [ˌɪndɪs'krɪmɪnətli] *adverb*

indispensable [ˌɪndɪs'pensəbl] *adjective*

monolingual [ˌmɒnəʊ'liŋgwəl] *adjective*

monolith ['mɒnəlɪθ] *noun*

monologue , ['mɒnə,lɒg] *noun*

monocracy [mɒ'nɒkrəsi] *noun*

pizzicato [ˌpɪtsɪ'kɑ:təʊ] *adverb*

Optimism ['ɒptɪmɪzəm]

Insignificance [ˌɪnsɪg'nɪfɪkəns]

Futility [fju:'tɪlɪtɪ]

Grateful ['greɪtfʊl]

Omission [əʊ'mɪʃən]

to destroy [dɪs'trɔɪ]

Mixture ['mɪkstʃə]

to issue ['ɪʃu:]

to give [gɪv]

to publish ['pʌblɪʃ]

to summer ['sʌmə]

convention [kən'venʃən]

conventional [kən'venʃənəl]

leaders ['li:dərz]

for example, for

screen [skri:n]

cover ['kʌvəʃ]

truthful ['tru:θfʊl]

to sacrifice ['sækrɪfaɪs]

colleague ['kɒli:g]

activate ['æktɪveɪt], animated ['ænɪmeɪt]

break [breɪk] = pause [pɔ:z] = rest

champion ['tʃæmpjən]

classified ['klæsɪfaɪd] adj

clean [kli:n] = neat [ni:t] = tidy ['taɪdɪ]

client ['klaɪənt]

coldness ['kɒldnɪs] n

colleague ['kɒli:g]

convocation [ˌkɒnvə'keɪʃən]

discussion [dɪs'kʌʃən] = debate [dɪ'beɪt]

distinction [dɪs'tɪŋkʃən]

divorce [dɪ'vɔ:ɪs]

eating ['i:tɪŋ] and drinking ['drɪŋkɪŋ]

equity ['ekwɪtɪ]

exaggeration [ɪgˌzædʒə'reɪʃən]

expense [ɪk'spens] = cost [kɒst] = charge [tʃɑ:dʒ]

fear [fɪəʃ] = terror ['terəʃ] n

formal [ˈfɔ:məl] adj

hall [hɔ:l]

happiness [ˈhæpɪnɪs] = joy

intention [ɪnˈtenʃən] = aim [eɪm] = purpose [ˈpɜ:pəs]

isolation [ˌaɪsəʊˈleɪʃən] = seclusion [sɪˈklu:ʒən]

justice [ˈdʒʌstɪs] = instalment, installment US [ɪnˈstɔ:lmənt]

lightness [ˈlaɪtnɪs] n

pave [peɪv], pave the way

pavement [ˈpeɪvmənt], the pavement

pavilion [pəˈvɪlɪən], n

pay [peɪ], the pay, to pay

peaceful [ˈpi:sfʊl] adj, peaceable [ˈpi:səbl] adj

peak [pi:k], con trough [trɒf]

practical [ˈpræktɪkəl]

practice [ˈpræktɪs], the practice

practise, practice US [ˈpræktɪs]

sacrifice [ˈsækrɪfaɪs]

sensation [senˈseɪʃən], feeling [ˈfi:lɪŋ] n

stipulation [ˌstɪpjʊˈleɪʃən], condition

suspect [ˈsʌspekt]

to disturb [dɪsˈtɜ:b] = to trouble [ˈtrʌbl]

to divide, to split

to increase [ɪn'kri:əs, 'ɪnkri:əs], to grow [grəʊ]

to spread [spred], to diffuse [dɪ'fju:z]

to sweeten ['swi:tən]

tribunal [traɪ'bjʊ:nl] noun

variable ['vɛəriəbl] = changeable ['tʃeɪndʒəbl]

execution [ˌeksɪ'kju:ʃən]

lie [laɪ]

garden ['gɑ:dn]

war [wɔ:r]

protest ['prəʊtest] : to accept an insult without protest.

relevance ['reləvəns]

relevance ['reləvəns], **relevancy** ['reləvənsɪ] noun

sanctify ['sæŋktɪfaɪ] transitive verb

sanctimoniously [ˌsæŋktɪ'məʊniəsli] adverb

tachyon ['tækɪ,ɒn] noun

Problem 5. Read correctly the following words.

visibility [ˌvɪzɪ'bɪlɪtɪ] noun

compute [kəm'pjʊt] transitive verb

computer [kəm'pjʊ:tər]

computing [kəm'pjʊ:tɪŋ]

cooperation [kəʊ,ɒpə'reɪʃən]

deferential [ˌdefə'renʃəl] adj

estimation [ˌestɪˈmeɪʃən] **noun**

liberation [ˌlɪbəˈreɪʃən]

pavilion [pəˈvɪlɪən]

reason [ˈriːzn]

reassurance [ˌriːəˈʃʊərəns]

purism [ˈpjʊərɪzəm]

steed [stiːd] **noun**

steeplechase [ˈstiːpltʃeɪs]

telson [ˈtelsən]

lionization [ˌlaɪənəɪˈzeɪʃən]

Problem 6. Read correctly the following words

[welθ]	[gʊd]	[ˈmɔːnɪŋ]	[wʊd]	[kʊd]
[saɪˈteɪʃən]	[treɪn]	[ˈmʌðər]	[ˈmɑːkɪt]	[mæp]
[rɪˈgretəblɪ]	[ˈkæmərə]	[məˈʃiːn]	[wʊlf]	[ˌɪntrəˈdʌkʃən]
[ˌredʒɪˈstreɪʃən]	[ˈspiːʃəs]	[ˈdʒænjʊəri]	[ɪnˈvæljʊəbl]	[ˌɪntɪˈfaɪdə]
[ˈklɪəli]	[ˈspektrəl]	[brʌʃ]	[ˈdʌbl]	[dɔːr]
[əʊˈmɪt]	[spuːn]	[ˈdɒləʳ]	[dɒg]	[ˈɒrɪndʒ]
[ˈsekʃən]	[fəʊn]	[ˌsɪtʃʊˈeɪʃən]	[ˈsɪkstɪ]	[ˈsevn]
[ˌɪnkəˈrekt]	[fəʊˈnetɪk]	[ˈsɪŋɡl]	[saɪz]	[ˈsɪɡnl]
[ˈvɜːgjuːl]	[ˈpeɪʃənt]	[ˈeɪnʃənt]	[əˈpɪnjən]	[əˈfɪʃəl]
[pəˈræmɪtər]	[ˈnəʊweər]	[ˈnʌmbər]	[ˈeni]	[ˈplʌnɪʃ]
[ɪmˈpɔːtənt]	[faʊnd]	[həʊp]	[ˈɒnɪst]	[haɪd]
[ˈɪntrɪkɪt]	[haɪt]	[hɑːt]	[welθ]	[ˈhɑːdlɪ]
[grəʊθ]	[ˈhæbɪt]	[graʊnd]	[ˈgreɪtnɪs]	[griːs]
[ˈɪntrɪst]	[gəʊl]	[ɪnˈtrænsɪtɪv]	[gəʊ]	[ˌkɒmbɪˈneɪʃən]
[hæv]	[ˈprezɪdənt]	[ˈkemɪstrɪ]	[ˈpɜːsnl]	[ˌɪndɪˈvɪdʒʊəl]
[teɪk]	[ˈævərɪdʒ]	[təˈgeðər]	[ˈpɒlɪtɪks]	[ɪnˈtɑːtl]
[ˈfɜːðər]	[mʌnθ]	[ˈelɪmənt]	[ˌjuːnɪˈvɜːsɪtɪ]	[ɪkˈstɛnʃən]

[jʊə'self]	[mɑ:tʃ]	['pleɪɪŋ]	[fi:ld]	[rɪ'sɜ:tʃ]
[ɪn'dʒɔɪ]	[bɔ:l]	['laɪbrəri]	['tɪkɪt]	['sɪnəmə]
['mɒsmənt]	['welkəm]	[sə'saɪətɪ]	[sneɪk]	[smɔ:l]
[ɪ'nʌf]	[tə'deɪ]	[təʊ]	['ɜ:dʒənt]	[ti:tʃ]

You should pronounce the **stress '** and the **vowels ɜ: i: ə u: ...**very well.

Part 3. Some words used in Computer Science

Read correctly the following words which have relation with computer science.

ability [ə'bɪlɪtɪ]

abstraction [æb'strækʃən]

according [ə'kɔ:dɪŋ]

achieve [ə'tʃi:v]

action ['ækʃən]

adapt [ə'dæpt]

agent ['eɪdʒənt]

algorithm ['ælgə,rɪðəm]

antivirus

approach [ə'prəʊtʃ]

architecture ['ɑ:kɪtektʃə]

artifact ['ɑ:tɪfækt] **noun**

artifact ['ɑ:tɪfækt] **noun**

artificial [ɑ:tɪ'fɪʃəl]

authors ['ɔ:θəʊz]

automation [ˌɔːtə'meɪʃən] *noun*

autonomous [ɔː'tɒnəməs] *adjective*

autonomously [ɔː'tɒnəməsli] *adverb*

backup ['bækʌp]

behavior ^{US} [bi'heɪvjər]

belief [bi'liːf]

binary ['baɪnəri]

BIOS. basic input output system

block [blɒk]-**based** [beɪst]

Boolean ['buːliən] *adjective*

browser ['braʊzər] *noun*

bug [bʌg]

bugs

byte [baɪt] *noun*

capsule ['kæpsjuːl]

category ['kætɪgəri]

central ['sentrəl]

challenge ['tʃælɪndʒ]

character set

character style

character user interface

charge [tʃɑːdʒ]

checksum

Cheese worm

cloud [klaʊd]

code [kəʊd], code name

cognitive ['kɒgnɪtɪv]

cognitive science

collective [kə'lektɪv]

comb [kəʊm]

command [kə'mɑ:nd]

compatibility and portability

component [kəm'pəʊnənt]

compression [kəm'preʃən]

computation [ˌkɒmpjʊ'teɪʃən]

computational artifact [ˌkɒmpjʊ'teɪʃənɪ]

computational thinking ['θɪŋkɪŋ]

computer [kəm'pjʊ:tər], computer system

concept ['kɒnsept]

conditional [kən'dɪʃənɪ]

context ['kɒntekst]

contour ['kɒntʊər]

control [kən'trəʊl]

cooperation [kəʊ,ɒpə'reɪʃən]

CPM [ˌsiːpiː'em] **noun**, abbreviation of **critical path method**

CPU [ˌsiːpiː'juː] **Computing** abbreviation of **central processing unit**

create [kriː'eɪt]

data ['deɪtə]

debugging [diː'bʌgɪŋ] **noun**

decision [dɪ'sɪʒən]

decompose [ˌdiːkəm'pəʊz]

decryption

deployment [dɪ'plɔɪmənt] **noun**

derive [dɪ'raɪv]

description [dɪs'krɪpʃən]

design [dɪ'zaɪn]

designate ['deɪzɪneɪt]

destroy [dɪs'trɔɪ]

diagram ['daɪəgræm]

digital ['dɪdʒɪtəl]

directly [dɪ'rektli]

divide [dɪ'vaɪd]

dynamic [daɪ'næmɪk]

e-books and digital libraries

echo ['ekəʊ]

education in the computer field

e-government

element ['elɪmənt] noun

embed , imbed [ɪm'bed]

emergence [ɪ'mɜːdʒəns]

Emoticon [ɪ'məʊtɪkən] noun Computing

emulation [ˌemjʊ'leɪʃən]

encryption [ɪn'krɪpʃən] noun Computing,
Telecommunications

enforcement [ɪn'fɔːsmənt]

environment [ɪn'vaɪərənmənt]

epoch ['iːpɒk]

event [ɪ'vent]

evolve [ɪ'vɒlv]

exec [ɪg'zek]

execution [ˌeksɪ'kjuːʃən]

exit ['eksɪt]

figure ['fɪgəʃ]

filter ['fɪltəʃ]

filter ['fɪltəʃ]

flexibility [ˌfleksɪ'bɪlɪtɪ] noun

FORTRAN, Fortran ['fɔːtræn] noun

framework ['freɪmwɜːk]

function ['fʌŋkʃən]

future ['fju:tʃəʳ]

generation [ˌdʒenə'reɪʃən]

Globalization [ˌglɒsbəlaɪ'zeɪʃən] **noun**

goal [gəʊl]

GPS [dʒi:pi:'es] abbreviation of **global positioning system**

graphics card

graphics formats, EPS, GIF

green PC

grid [grɪd]

grid computing

groupware ['gru:pweəʳ]

hacker ['hækəʳ] **Computing**

hacking ['hækɪŋ]

hacking ['hækɪŋ] **noun** **Comput**

hardware ['hɑ:dweəʳ]

help systems

hierarchy ['haɪəɹɑ:kɪ] **noun**

homogenous [hə'mɒdʒɪnəs] **adjective**

http [ˌeɪtʃti:ti:'pi:], abbreviation of **hypertext transfer protocol** : **http**

human ['hju:mən]

hybrid ['haɪbrɪd]

Hytime

Hz Radio, abbreviation of **hertz**

identical [aɪ'dentɪkəl]

image ['ɪmɪdʒ]

IMAP

independent [ˌɪndɪ'pendənt]

input ['ɪnpʊt]

insolvency [ɪn'sɒlvənsɪ] **noun**

insolvency [ɪn'sɒlvənsɪ] **noun**

instruction [ɪn'strʌkʃən]

intelligence [ɪn'telɪdʒəns]

intelligent [ɪn'telɪdʒənt]

interaction [ˌɪntər'ækʃən] **noun**

Internet ['ɪntə.net]

introduction [ˌɪntrə'dʌkʃən]

involve [ɪn'vɒlv]

iterative ['ɪtərətɪv]

Language translation software

laptop computer

layer ['leɪə]

library, program

linguistics and computing

Linux

list processing

local area network (LAN)

logic ['lɒdʒɪk]

long [lɒŋ]

loop [lu:p]

lossless

lossy

machines [mə'ʃi:nz]

management ['mænɪdʒmənt]

management ['mænɪdʒmənt]

MAS, [em, eɪ, es]

matching ['mætʃɪŋ]

memory ['memərɪ]

MMC. Acronym for Microsoft Management Console.

model ['mɒdl]

model ['mɒdl]

mouse [maʊs]

multi... ['mʌltɪ] *prefix*

multi-agent, multiagent

netiquette ['netɪket] *noun*

network ['netwɜ:k]

neural ['njʊərəl]

offer ['ɒfə]

ontologies and data models

open-source movement

orientation [ˌɔːrɪən'teɪʃən]

OS [əʊ'es], **Computing** abbreviation of **operating system**

outside ['aʊt'saɪd]

over ['əʊvə]

packet ['pækɪt]

paradigm ['pærədəɪm]

parallelism ['pærələlɪzəm]

parameter [pə'ræmɪtə]

pattern ['pætən]

persistence [pə'sɪstəns] , **persistency** [pə'sɪstənsɪ] **noun**

piracy ['paɪərəsɪ]

popular culture and computing

population [ˌpɒpjʊ'leɪʃən]

postscript ['pəʊsskrɪpt] **noun**

prefer [prɪ'fɜː]

printer ['prɪntə]

privacy in the digital age

procedure [prə'sɪdʒə] **noun**

processor ['prəʊsesə] **noun**

programming ['prəʊgræmɪŋ]

programming as a profession

projection [prə'dʒekʃən]

prototype ['prəʊtəʊtaɪp] **noun**

pseudocode ['sjuːdəʊs]

reactive [ri:'æktɪv] **adjective**

reality [rɪ'ælɪtɪ]

reflex ['ri:fleks]

region ['ri:dʒən]

relationship [rɪ'leɪʃənʃɪp]

replace [rɪ'pleɪs]

request [rɪ'kwest]

requirement [rɪ'kwaɪəmənt]

response [rɪ'spɒns]

RGB

ROM [rɒm] **noun** **Computing** abbreviation of **Read-Only-Memory**

rule [ru:l]

rule [ru:l]

scale [skeɪl]

security [sɪ'kjʊərɪtɪ]

segmentation [ˌsegmən'teɪʃən]

sensor ['sensəʳ] **noun** *détecteur*^m

server ['sɜ:vəʳ] **noun**

simple ['sɪmpl]

simplicity [sɪm'plɪsɪtɪ]

simulate ['sɪmjʊleɪt]

simulation [ˌsɪmjʊ'leɪʃən]

situation [ˌsɪtjʊ'eɪʃən]

SMTP

social ['səʊʃəl]

software ['sɒft,weəʳ] **Computing**

some [sʌm]

storage ['stɔ:ɹɪdʒ]

string [strɪŋ]

structure ['strʌktʃəʳ]

subroutine ['sʌbru:ti:n] **noun** **Computing** **sous-programme**^m

subsection ['sʌb,sekʃən]

summary ['sʌməɹɪ]

switch [swɪtʃ]

system ['sɪstəm]

tablet ['tæblɪt] **noun**, graphics tablet

term [tɜ:m]

theoretical [θɪə'retɪkəl]

thinking ['θɪŋkɪŋ]

time [taɪm]

topology [tə'pɒlədʒɪ]

track [træk]

transform [træns'fɔ:m]

troubleshooting ['trʌbl,ʃu:tɪŋ]

uninstall [ʌnɪn'stɔ:l] deinstall

valid ['vælɪd]

variable ['vɛərɪəbl]

whole, the whole system [həʊl] adjective

zip [zɪp]

Read correctly the following words

down [daʊn]	downpipe ['daʊn,paɪp]	downsizing ['daʊnsaɪzɪŋ]	during ['djʊərɪŋ]
weather ['weðər]	bureau ['bjʊərəʊ]	affair [ə'feər]	there [ðeər]
problem ['prɒbləm]	working ['wɜ:kɪŋ]	work [wɜ:k]	Koran [kɒ'rɑ:n]

inhuman [ɪn'hju:mən]	Muslim ['mʊzlɪm]
improve [ɪm'pru:v]	improvement [ɪm'pru:vmənt]
incite [ɪn'saɪt]	optimism ['ɒptɪmɪzəm]
inauguration [ɪ,nɔ:ɡjʊ'reɪʃən]	confusion [kən'fju:ʒən]
incorrect [ɪn'kɒ'rekt]	Islamic [ɪz'læmɪk]
frank [fræŋk], adj	goods [ɡʊdz]
impious ['ɪmpɪəs]	suffering ['sʌfərɪŋ]
variety [və'reɪtɪ], varieties	pain [peɪn]
organization [ɔ:ɡənəɪ'zeɪʃən]	affection [ə'fekʃən]
thesis ['θɪ:sɪs], pl theses ['θɪ:sɪz]	essential [ɪ'senʃəl]
To face [feɪs]	fundamental [ˌfʌndə'mentl]
whereas [weər'æz]	envious ['envɪəs]
vocation [vəʊ'keɪʃən]	edge [edʒ]
venerate ['venərəɪt]	necessity [nɪ'sesɪtɪ]

rude [ru:d]	perfection [pə'fekʃən]
prepare [prɪ'peəɹ]	To converse [kən'vɜ:s]
defeat [di'fi:t]	vicious ['viʃəs]
To discipline ['dɪsɪplɪn]	generosity [ˌdʒenə'rɒsɪtɪ]
To menace ['menɪs]	ignore [ɪg'nɔ:ɹ]
fictitious [fɪk'tɪʃəs]	fill [fɪl]
adviser, advisor [əd'vaɪzəɹ]	imitation [ˌɪmɪ'teɪʃən], tradition [trə'dɪʃən]
beneficial [ˌbenɪ'fɪʃəl]	impose [ɪm'pəʊz]
childbirth ['tʃaɪldbɜ:θ]	movement ['mu:vmənt]
ministry ['mɪnɪstrɪ]	necessary ['nesɪsərɪ], adj, n
needless ['ni:dlɪs], adj	pharmacy ['fɑ:məsɪ]
poverty ['pɒvətɪ]	probable ['prɒbəbl]
real [rɪəl]	personality [ˌpɜ:sən'neɪlɪtɪ]
crowd [kraʊd], n	funeral ['fju:nərəl]
personally ['pɜ:snəlɪ]	silence ['saɪləns]
silk [sɪlk]	sensitivity [ˌsensɪ'tɪvɪtɪ]
smooth [smu:ð]	thinking ['θɪŋkɪŋ]
stipulate ['stɪpjʊleɪt]	repetition [ˌrepɪ'tɪʃən]
to educate ['edʒʊkeɪt]	to evacuate [ɪ'vækjʊeɪt], to empty ['emptɪ]
successful [sək'sesfʊl]	to benefit ['benɪfɪt]
to efface [ɪ'feɪs]	maintenance ['meɪntɪnəns]
immediately [ɪ'mɪdiətɪ]	morale [mɒ'rɑ:l], n
leadership ['li:dəʃɪp], n	love [lʌv], affection [ə'fekʃən]
literally ['lɪtərəlɪ]	legal ['li:gəl]
insignificance [ˌɪnsɪg'nɪfɪkəns]	illegal [ɪ'li:gəl]
honour, honor US ['ɒnəɹ], nobility [nəʊ'bɪlɪtɪ], dignity ['dɪgnɪtɪ]	honourable, honorable US ['ɒnərəbl]
hardness ['hɑ:dnɪs]	hint [hɪnt], allusion [ə'lu:ʒən]
to clarify ['klærɪfaɪ]	glorious ['glɒ:rɪəs], great [greɪt]
fisherman ['fɪʃəmən]	fight [faɪt], n, v
feeble ['fi:bl]	feast [fi:st]
fear [fɪəɹ], n, v	fabulous ['fæbjʊləs]
employ [ɪm'plɔɪ]	extinction [ɪk'stɪŋkʃən]
educated ['edʒʊkeɪtɪd]	emotion [ɪ'məʊʃən]
deception [di'sepʃən] definitively [di'fɪnɪtɪvɪ]	disarray [ˌdɪsə'reɪ]
off [ɒf]	death [deθ]
crisis ['kraɪsɪs]	criterion [kraɪ'tɪərɪən]
complement ['kɒmplɪmənt], supplement ['sʌplɪmənt]	courage ['kʌrɪdʒ]

promise ['prɒmɪs], n, v	spoil [spɔɪl]
slim [slɪm]	tidy ['taɪdɪ]
necessarily ['nesɪsərɪli], adv	to dine [daɪn]
to clap the hands [klæp]	to evaporate [ɪ'væpəreɪt]
to deprive of [dɪ'praɪv]	to evolve [ɪ'vɒlv]
to dilate [daɪ'leɪt]	to help [help] to assist [ə'sɪst]
to lead [li:d]	to liken ['laɪkən]
to suffer ['sʌfə]	unluckily [ʌn'lʌkɪli]
unlucky [ʌn'lʌkɪ]	unreal ['ʌn'rɪəl]
untidy [ʌn'taɪdɪ]	event [ɪ'vent]
witness ['wɪtnɪs]	register ['redʒɪstə]
denial [dɪ'naɪəl] noun	opinion [ə'pɪnjən]
liberty ['lɪbətɪ]	view [vju:]
to be the chief [tʃi:f]	wide [waɪd]

Remark. The following words are used in journals.

accommodation [ə,kɒmə'deɪʃən]	archive ['ɑ:kɑɪv]
affiliation [ə,fɪlɪ'eɪʃən]	conference ['kɒnfərəns]
expert ['ekspɜ:t]	format ['fɔ:mæt]
extended [ɪk'stendɪd]	participant [pɑ:'tɪsɪpənt]
fee [fi:]	promotion [prə'məʃjən]
provide [prə'vaɪd]	registration [ˌredʒɪ'streɪʃən]
reviewer [rɪ'vju:ə]	search [sɜ:tʃ]
theoretical [θɪə'retɪkəl]	version ['vɜ:ʃən]
privacy ['prɪvəsɪ]	consult [kən'sʌlt]
proceed [prə'si:d]	proceed [prə'si:d]
editor ['edɪtə]	manuscript ['mænʃskrɪpt]
update [ʌp'deɪt]	backlog ['bæklɒg]
significant [sɪg'nɪfɪkənt]	contribution [ˌkɒntrɪ'bju:ʃən]
assurance [ə'ʃʊərəns]	publish ['pʌblɪʃ]
Congress ['kɒŋɡres]	postage ['pəʊstɪdʒ]
per [pɜ:r]	issue ['ɪʃu:]
postal address ['pəʊstəl] [ə'dres]	rate [reɪt]
institutional [ˌɪnstɪ'tju:ʃən]	register ['redʒɪstə]
conditional [kən'dɪʃən]	rational ['ræʃən]
professional [prə'feʃən]	rationalization [ˌræʃnəlaɪ'zeɪʃən]

Part 4. What does exist in general mathematics?

• NUMBERS

Sets. Real numbers. Decimal representation of real numbers. Geometric representation of real numbers. Operations with real numbers. Inequalities. Absolute value of real numbers. Exponents and roots. Logarithms. Axiomatic foundations of the real number system. Point sets, intervals. Countability. Neighborhoods. Limit points. Bounds. Bolzano- Weierstrass theorem. Algebraic and transcendental numbers. The complex number system. Polar form of complex numbers. Mathematical induction.

• SEQUENCES

Definition of a sequence. Limit of a sequence. Theorems on limits of sequences. Infinity. Bounded, monotonic sequences. Least upper bound and greatest lower bound of a sequence. Limit superior, limit inferior. Nested intervals. Cauchy's convergence criterion. Infinite series.

• FUNCTIONS, LIMITS, AND CONTINUITY

Functions. Graph of a function. Bounded functions. Monotonic functions. Inverse functions. Principal values. Maxima and minima. Types of functions. Transcendental functions. Limits of functions. Right- and left-hand limits. Theorems on limits. Infinity. Special limits. Continuity. Right- and left-hand continuity. Continuity in an interval. Theorems on continuity. Piecewise continuity. Uniform continuity.

• DERIVATIVES

The concept and definition of a derivative. Right- and left-hand derivatives. Differentiability in an interval. Piecewise differentiability. Differentials. The differentiation of composite functions. Implicit differentiation. Rules for differentiation. Derivatives of elementary functions. Higher order derivatives. Mean value theorems. L'Hospital's rules. Applications.

• INTEGRALS

Introduction of the definite integral. Measure zero. Properties of definite integrals. Mean value theorems for integrals. Connecting integral and differential calculus. The fundamental theorem of the calculus. Generalization of the limits of integration. Change of variable of integration. Integrals of elementary functions. Special methods of integration. Improper integrals. Numerical methods for evaluating definite integrals. Applications. Arc length. Area. Volumes of revolution.

• PARTIAL DERIVATIVES

Functions of two or more variables. Three-dimensional rectangular coordinate systems. Neighborhoods. Regions. Limits. Iterated limits. Continuity. Uniform continuity. Partial derivatives. Higher order partial derivatives. Differentials. Theorems on differentials. Differentiation of composite functions. Euler's theorem on homogeneous functions. Implicit functions. Jacobians. Partial derivatives using Jacobians. Theorems on Jacobians. Transformation. Curvilinear coordinates. Mean value theorems.

• VECTORS

Vectors. Geometric properties. Algebraic properties of vectors. Linear independence and linear dependence of a set of vectors. Unit vectors. Rectangular (orthogonal unit) vectors. Components of a vector. Dot or scalar product. Cross or vector product. Triple products. Axiomatic approach to vector analysis. Vector functions. Limits, continuity, and derivatives of vector functions. Geometric interpretation of a vector derivative. Gradient, divergence, and curl. Formulas involving r . Vector interpretation of Jacobians, Orthogonal curvilinear coordinates. Gradient, divergence, curl, and Laplacian in orthogonal curvilinear coordinates. Special curvilinear coordinates.

• APPLICATIONS OF PARTIAL DERIVATIVES

Applications to geometry. Directional derivatives. Differentiation under the integral sign. Integration under the integral sign. Maxima and minima. Method of Lagrange multipliers for maxima and minima. Applications to errors.

• MULTIPLE INTEGRALS

Double integrals. Iterated integrals. Triple integrals. Transformations of multiple integrals. The differential element of area in polar coordinates, differential elements of area in cylindrical and spherical coordinates.

• LINE INTEGRALS, SURFACE INTEGRALS, AND INTEGRAL THEOREMS

Line integrals. Evaluation of line integrals for plane curves. Properties of line integrals expressed for plane curves. Simple closed curves, simply and multiply connected regions. Green's theorem in the plane. Conditions for a line integral to be independent of the path. Surface integrals. The divergence theorem. Stoke's theorem.

• INFINITE SERIES

Definitions of infinite series and their convergence and divergence. Fundamental facts concerning infinite series. Special series. Tests for convergence and divergence of series of constants. Theorems on absolutely convergent series. Infinite sequences and series of

functions, uniform convergence. Special tests for uniform convergence of series. Theorems on uniformly convergent series. Power series. Theorems on power series. Operations with power series. Expansion of functions in power series. Taylor's theorem. Some important power series. Special topics. Taylor's theorem (for two variables).

• **IMPROPER INTEGRALS**

Definition of an improper integral. Improper integrals of the first kind (unbounded intervals). Convergence or divergence of improper integrals of the first kind. Special improper integrals of the first kind. Convergence tests for improper integrals of the first kind. Improper integrals of the second kind. Cauchy principal value. Special improper integrals of the second kind. Convergence tests for improper integrals of the second kind. Improper integrals of the third kind. Improper integrals containing a parameter, uniform convergence. Special tests for uniform convergence of integrals. Theorems on uniformly convergent integrals. Evaluation of definite integrals. Laplace transforms. Linearity. Convergence. Application. Improper multiple integrals.

• **FOURIER SERIES**

Periodic functions. Fourier series. Orthogonality conditions for the sine and cosine functions. Dirichlet conditions. Odd and even functions. Half range Fourier sine or cosine series. Parseval's identity. Differentiation and integration of Fourier series. Complex notation for Fourier series. Boundary-value problems. Orthogonal functions.

• **FOURIER INTEGRALS**

The Fourier integral. Equivalent forms of Fourier's integral theorem. Fourier transforms.

• **GAMMA AND BETA FUNCTIONS**

The gamma function. Table of values and graph of the gamma function. The beta function. Dirichlet integrals.

• **FUNCTIONS OF A COMPLEX VARIABLE**

Functions. Limits and continuity. Derivatives. Cauchy-Riemann equations. Integrals. Cauchy's theorem. Cauchy's integral formulas. Taylor's series. Singular points. Poles. Laurent's series. Branches and branch points. Residues. Residue theorem. Evaluation of definite integrals.

Part 5. Basic mathematical arguments

1. For example, let $m = 15$ and $a = 7$.
2. Since $f(x) = y$, Euler's Theorem tells us that
3. We can compute the order as follows:
4. and so the order of E is 4 .
5. We shall give a second proof of Euler's theorem and its corollaries.
6. We begin with some simple observations about functional analysis.
7. We define the order of a group as the cardinality of the group.

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1. **(Lagrange's theorem)** If G is a finite group and H is a subgroup of G , then the order of H divides the order of G .
2. The map $f: X \rightarrow aX$ defined by $f(x) = ax$ is a bijection.
3. In particular, we see that
4. The arithmetic function $\omega(n)$ counts the number of distinct prime divisors of the positive integer n , that is,

$$\omega(n) = \sum_{p|n} 1.$$

1. where b is the positive real number defined by (3.2).
2. **Applying** Chebyshev's theorem (Theorem 3.4), we get
3. we obtain.
4. From Theorem 3.5, we obtain
5. For $x \geq 2$,
6. By Theorem 8.9,

1. This completes the proof.
2. Let S be a finite set of integers, and let f be a real-valued function defined on S .
3. For every $\delta > 0$, the number of integers $n \leq x$ such that
4. Let S be the set of positive integers n not exceeding x .
5. Applying Chebyshev's inequality,

1. We use Theorem 8.9 and Theorem 8.10 to evaluate this sum as follows:
2. Prove that there exists a constant b such that for $x \geq 2$,
3. There are many beautiful open problems about prime numbers. [Here are some examples.](#)
4. Do there exist infinitely many primes p of the form $n^2 + 1$?
5. A linear space over the field \mathbf{K} is a non-empty set \mathbf{V} of vectors with a binary operation (*addition*) and a scalar multiplication.

1. Let $G \subset E$ be open.
2. we denote by $D\{\alpha\}$ the partial derivative
3. These spaces play a central role in our work
4. A subset M of the linear space V is a subspace of V if it is closed under the linear operations.
5. We have three chains of subspaces given by :
6. We let M be a subspace of V and construct a corresponding quotient space.
7. T is continuous at x **if and only if**

1. We shall define addition of cosets by adding a corresponding pair of representatives and similarly define scalar multiplication. It is necessary to first verify that this definition is **unambiguous**.
2. **Proof** The proof is analogous to that of **(2.1.1)**.
3. Recalling the definition of "norm"
4. this gives the inequality (5.2).
5. Now, putting $\lambda = 1$ in (5.3) and using (5.2) on the right yields

1. The first two axioms of norm, namely that
2. for all $\lambda \in \mathbb{R}$ and all $f \in V$,
3. We have thus shown the following result.
4. follow directly from **(9.1)** and from the last three axioms of inner product stated in Definition **5.1**.
5. the set $C[a, b]$ of continuous real-valued functions defined on the closed interval $[a, b]$ is an inner product space

1. The problem of best approximation in the 2 -norm can be formulated as follows:
2. such p is called a polynomial of best approximation of degree n to the function f in the 2 -norm on (a, b) .
3. The existence and uniqueness of p will be shown in Theorem **4.2**. However, we shall first consider some simple examples.
4. Suppose that $\varepsilon > 0$ and let
5. We shall construct the polynomial of best approximation of degree n in the 2 -norm,

1. In the previous section we described a method for constructing the polynomial of best approximation p to a function f in the 2-norm;
2. with the inner product $(.,.)$ defined by
3. Given a weight function w , defined, positive, continuous and integrable on the interval (a, b) ,
4. *Next, we show that a system of orthogonal polynomials exists on any interval (a, b) and for any weight function w which satisfies the conditions in Definition 5.1.*

1. Let us now define the polynomial
2. It then follows that
3. where we have used the orthogonality of the sequence
4. This procedure for constructing a system of orthonormal polynomials is usually referred to as Gram–Schmidt **orthonormalisation**.

1. and therefore,
2. Clearly,
3. for any pair of nonnegative integers m and n .
4. we recall the definition of the inner product $(.,.)$
5. Such a system of polynomials is said to be orthonormal.
6. Using this result with $k = 1$, we obtain

1. **By substituting this into (1.6) we get**
2. Thus, in particular, **Thus = Ainsi**
3. provided that
4. **On taking the (natural) logarithm of each side in the last inequality, we find that**
5. Now we can return to Example 1.2 to answer the question posed there about the maximum number of iterations.

1. **Consider the problem of determining the solutions of the equation $f(x) = 0$,**
2. The function f is monotonic increasing for positive x and monotonic decreasing for negative values of x . Moreover,
3. The equation $f(x) = 0$ may be written in the equivalent form :
4. **On the other hand,**

1. Evidently the given equation may be written in many different forms, leading to iterations with different properties.
2. **In the previous section we saw**
3. In fact, by applying the Contraction Mapping Theorem on an arbitrary bounded closed interval $[0, M]$ where
4. **Convergence of Newton's method :**
5. Using Newton's method to solve a nonlinear equation :

1. Suppose further that there exists a positive constant A such that
2. this shows that
3. According to Definition 1.7, implies
4. Suppose that the function f satisfies the conditions of Theorem 1.8 and also that there exists a real number $X, X > \xi$, such that :
5. It follows from (1.23) that
6. Choosing $\varepsilon = \alpha$ we see that

From (1.25) and using the Mean Value Theorem (Theorem A.3) together with the fact that $f(\xi) = 0$, we obtain

It follows from . . . that . . .

We deduce from . . . that . . .

Conversely, . . . implies that . . .

Equality (1) holds, by Proposition 2.

By Definition 2.1, . . .

The following statements are equivalent.

Thanks to . . . , the properties . . . and . . . of . . . are equivalent to each other.

. . . has the following properties.

Theorem 1 holds unconditionally.

This result is conditional on Axiom A.

. . . is an immediate consequence of Theorem 3.

Note that . . . is well-defined, since . . .

As . . . satisfies . . . , formula (1) can be simplified as follows.

We conclude (the argument) by combining inequalities (2) and (3).

(Let us) denote by X the set of all . . .

Let X be the set of all . . .

Recall that . . . , by assumption.

It is enough to show that . . .

We are reduced to proving that . . .

The main idea is as follows.

We argue by contradiction. Assume that . . . exists.

The formal argument proceeds in several steps.

Consider first the special case when . . .

The assumptions . . . and . . . are independent (of each other), since . . .

. . . , which proves the required claim.

We use induction on n to show that . . .

On the other hand, . . .

. . . , which means that . . .

In other words, . . .

The Jacobi method

We give a general [view](#) about Jacobi's method, we [describe](#) in this section the method [was discovered](#) by Jacobi in 1846 and can be used [iteratively](#) to compute all the eigenvalues and eigenvectors of a real symmetric matrix.

Uniqueness of the distance from a point to a convex set: *the geometric meaning*

The general case is more [complicated](#); we start with a more general problem. Let M be a [convex](#) closed set in H . Denote the distance of x to the set M with $\rho(x, M)$. Then there exists a unique $y \in M$ such that $\rho(x, M) = \|x - y\|$ (the distance is [achieved](#) at the unique element $y \in M$).

The definite article (and its absence)

Measure theory, [théorie de la mesure](#)

number theory, [théorie des nombres](#)

Chapter one, [le chapitre un](#)

Equation (7), [l'équation \(7\)](#)

Harnack's inequality, [l'inégalité de Harnack](#)

the Harnack inequality

The Riemann hypothesis, [l'hypothèse de Riemann](#)

the Poincaré conjecture, [la conjecture de Poincaré](#)

Minkowski's theorem, [le théorème de Minkowski](#)

the Minkowski theorem

the Dirac delta function, [la fonction delta de Dirac](#)

Dirac's delta function

the delta function, [la fonction delta](#)

Part 6. On the correct pronunciation of certain Mathematical statements

6.1. Inequalities

$x > y$	x is greater than y (x is larger than y).
$x \geq y$	x is greater (than) or equal to y.
$x < y$	x is smaller than y.
$x \leq y$	x is smaller (than) or equal to y.
$x > 0$	x is positive.
$x \geq 0$	x is positive or zero; x is non-negative.
$x < 0$	x is negative.
$x \leq 0$	x is negative or zero.

6.2. Operators and calculus

$\lim_{n \rightarrow \infty} x_n = 0$	<ol style="list-style-type: none"> 1. The limit of x, n as n tends to infinity equals (is equal to) zero. 2. The sequence x, n tends to zero as n tends to infinity. 3. x, n tends to zero as n tends to infinity. 4. x, n tends to zero as n approaches (as n goes) to infinity.
---------------------------------------	--

$\frac{x}{y} = x \cdot (y^{-1})$,	x over y equals x times y minus one.
------------------------------------	--------------------------------------

$X \cup Y = \{x : x \in X \text{ or } x \in Y\}$.	X union Y. The union of X and Y equals the set of x such that x belongs big X or x belongs big Y
--	--

$2^x 3^y$	<p>two to the x times three to the y.</p> <p>two to the power of x times three to the power of y.</p>
-----------	---

$A = A^* \iff \forall (i, j) : a_{ij} = \overline{a_{ji}}$	The matrix A is Hermitian if and only if, for all i, j we have $a_{i,j}$ equals $a_{j,i}$ bar. The matrix A is equal to A star if and only if, for all i, j we have $a_{i,j}$ equals $a_{j,i}$ bar.
--	--

$S \Rightarrow T$	S implies T ; if S then T
$S \Leftrightarrow T$	S is equivalent to T ; S iff T

$(1 + 2)^{2+2}$	one plus two, all to the power of two plus two one plus two, all to the four
-----------------	---

x^2	x squared, n squared, α squared, B squared, ...
x^3	x cubed
x^n	x to the power of n , x to the n

$5 - 2 = 3$	Five minus two equals three
5^{-2}	five to the minus two
$x - 2$	x minus two

$\forall x \in A \dots$	for each [= for every] x in A ... for every x belongs to A
-------------------------	--

$\frac{1}{2}$	one half, one over two
$\frac{1}{3}$	one third, one over three
$\frac{1}{4}$	one quarter [= one fourth]
$\frac{1}{5}$	one fifth, one over five
$-\frac{1}{17}$	minus one seventeenth, minus one over seventeen
-0.067	minus nought point zero six seven
81.59	eighty-one point five nine

$-2.3 \cdot 10^6$ $= -2\,300\,000$	minus two point three times ten to the six minus two million three hundred thousand
---------------------------------------	--

$4 \cdot 10^{-3}$ $= 0.004 = 4/1000$	four times ten to the minus three four thousandths
---	---

$\{x \mid \dots\}$	the set of all x such that ...
--------------------	----------------------------------

$A \cup B$	the union of (the sets) A and B ; A union B
------------	---

$A \cap B$	the intersection of (the sets) A and B ; A intersection B
------------	---

$A \times B$	the product of (the sets) A and B ; A times B
--------------	---

$x, y \in A$	(both) x and y are elements of A ; ... lie in A ; ... belong to A ; ... are in A
--------------	---

$x, y \notin A$	(neither) x nor y is an element of A ; ... lies in A ; ... belongs to A ; ... is in A
-----------------	--

$A \cap B = \emptyset$	A is disjoint from B ; the intersection of A and B is empty.
------------------------	--

$x \in A$	x is an element of A x lies in A x belongs to A x is in A
-----------	--

$3 + 5 = 8$ three plus five equals [= is equal to] eight

$3 - 5 = -2$ three minus five equals [= ...] minus two

$3 \cdot 5 = 15$ three times five equals [= ...] fifteen

$(2 - 3) \cdot 6 + 1 = -5$ two minus three in brackets times six plus one equals minus five

$4!$ [= $1 \cdot 2 \cdot 3 \cdot 4$] four factorial.

$\frac{3}{5} = 0.6$	three divided by five equals zero point six.
$\exists x \in A \dots$	there exists [= there is] an x in A (such that) . . .
$\exists! x \in A \dots$	there exists [= there is] a unique x in A (such that) . . .
$\nexists x \in A \dots$	there is no x in A (such that) . . .

$\frac{3}{8}$	three eighths
---------------	---------------

$\frac{26}{9}$	twenty-six ninths
----------------	-------------------

$-\frac{5}{34}$	minus five thirty-fourths
-----------------	---------------------------

-245	minus two hundred and forty-five
------	----------------------------------

$\frac{1-3}{2+4} = -\frac{1}{3}$	one minus three over two plus four equals minus one third.
----------------------------------	--

$x > 0 \wedge y > 0 \implies x + y > 0$	if both x and y are positive, so is $x + y$
---	---

$\nexists x \in \mathbf{Q} \quad x^2 = 2$	no rational number has a square equal to two
---	--

$\forall x \in \mathbf{R} \exists y \in \mathbf{Q} \quad x - y < 2/3$	for every real number x there exists a rational number y such that the absolute value of x minus y is smaller than two third.
---	---

$\sin(x)$	sine x
-----------	----------

$\cos(x)$	cosine x
-----------	------------

$\tan(x)$	tan x
-----------	---------

$\arcsin(x)$	arc sine x
$\arccos(x)$	arc cosine x
$\arctan(x)$	arc tan x
$\sinh(x)$	hyperbolic sine x
$\cosh(x)$	hyperbolic cosine x
$\tanh(x)$	hyperbolic tan x
$\sin(x^2)$	sine of x squared
$\sin(x)^2$	sine squared of x; sine x, all squared
$\frac{x+1}{\tan(y^4)}$	x plus one, all over over tan of y to the four
$3x - \cos(2x)$	three to the (power of) x minus cosine of two x
$\exp(x^3 + y^3)$	exponential of x cubed plus y cubed

$p \notin R.$	<p>p does not belong to (the set) R. p is not in R. p is not an element of R. p does not lie in R.</p>
---------------	---

$(x + y)z + xy$	x plus y in brackets times z plus x, y
-----------------	--

$x^2 + y^3 + z^5$	x squared plus y cubed plus z to the power of five.
-------------------	---

$A = a^2$	Capital a equals small a squared.
-----------	-----------------------------------

$\overline{1 - 2i} = 1 + 2i$	The complex conjugate of one minus two i equals one plus two i .
\overline{z}	One minus two i bar equals one plus two i .
	The conjugate of a complex number z .

$x \leq 0$: x is negative or zero.

$x < 0$: x is negative.

$x \leq y$: x is smaller or equal to y or x is smaller than or equal to y .

$x - y = x + (-y)$	x minus y is equal to x plus, minus y
--------------------	---

$ax^2 + 2hxy + by^2 = 0 \dots(*)$	We consider the equation star: a , x squared plus two h , x,y plus b (times), y , squared is equal to zero.
-----------------------------------	---

$B = A - (A - B) = A [I - A^{-1}(A - B)]$	I minus A minus one times] B equals A [(A minus B) minus (A minus B) equals A times
---	---

$\lim_{x \rightarrow 0} \frac{f''(x)}{F''(x)} = \lim_{x \rightarrow 0} \frac{-e^x}{4} = -\frac{1}{4}$	The limit as x tends to zero of f two primes of x over big f two primes of x is equal to the limit as x tends to zero of minus exponential x over four which is equal to minus one over four.
---	---

$u_{n_1}, u_{n_2}, u_{n_3}, \dots$	We consider the subsequence $u_{,n}$ one, $u_{,n}$, two, ... and so on.
------------------------------------	---

$A \sim B \implies e^A \sim e^B$	If A is similar to B , then exponential A is also similar to exponential B .
	A is similar to B , implies exponential A is similar to exponential B .

$r = \sqrt{x^2 + y^2}$	r equals the square root of x squared plus y squared.
------------------------	---

$x^n + y^n = z^n$	x to the n plus y to the n equals z to the n
-------------------	--

$(x + y)z + xy$	x plus y in brackets times z plus x y
-----------------	---

$cA = \{cx \mid x \in A\}$.	c, A equals the set c times x such that x belongs to A
------------------------------	--

$A_n = \{x \in A \mid x \leq n\}$	A, n equals to the set of x belongs to A such that x is less (than) or equal to n .
-----------------------------------	---

$f(x), G(x)$	f of x , big g of x or g of x
--------------	---

(a, b)	open interval a, b .
----------	------------------------

$[a, b]$	closed interval a, b .
----------	--------------------------

$(a, b]$	Half open interval a, b (open on the left, closed on the right)
----------	---

$[a, b)$	Half open interval a, b (open on the right, closed on the left).
----------	--

$x \leq y$	x is smaller (than) or equal to y . x is smaller (than) or equals y .
------------	--

f'	f dash; f prime; the first derivative of f
------	--

f''	f double dash; f double prime; the second derivative of f
-------	---

$f(3)$	the third derivative of f
--------	-----------------------------

$f^{(n)}$	the n-th derivative of f
-----------	----------------------------

$\frac{dy}{dx}$	d y by d x; the derivative of y by x
-----------------	--------------------------------------

$n \leq x < n + 1.$	n is less or equal to x which is strictly less than n plus one.
---------------------	---

$- x \leq x \leq x .$	Minus the absolute value of x is less or equal to x which is less or equal to the absolute value of x.
-------------------------	--

$ ab = a \cdot b .$	The absolute value of a, b is equal to the absolute value of a times the absolute value of b .
-------------------------	--

$b = x - y :$	b equals x minus y .
$a = x + y$	a equals x plus y .

$A \neq \emptyset$	A is different from the empty set. A is non-empty.
--------------------	---

$c = x \cdot y \cdot z$	c equals x times y times z
$c = x y z$	c equals x, y, z

$\sum_{k=1}^n cr^k, \quad n = 1, 2, \dots$	The sum for k from one to n of c times r to the power of k . The sum of c times r to the power of k , for k from one to n
--	--

$\left\ \frac{A^k}{k!} \right\ \leq \frac{\ A\ ^k}{k!}$	The norm of A to the power of k over k factorial is less or equal to the norm of A to the power of k over k factorial.
---	--

$\left \sum_{k=1}^n x_k \right \leq \sum_{k=1}^n x_k .$	<p>The absolute value of the sum for k from one to n of x,k is less or equal to the sum for k from one to n of the absolute value of x,k.</p>
--	---

$\lim_{x \rightarrow 1} f(x) = 2$	<p>The limit of f of x as x tends to one is equal to two.</p>
-----------------------------------	---

$a^{n+1} - b^{n+1} = (a - b) \cdot \sum_{k=0}^n a^k b^{n-k}, \quad n = 1, 2, \dots$	<p>to the power of n plus one, or to the n plus one.</p>
<p>a to the power of n plus one, minus b to the power of n plus one equals a minus b times the sum for k from zero to n, of a to the power of k, b to the power of n minus k, where n equals one, two, and so on.</p>	

x^{-1}	<p>x to the minus one x to the power of minus one. The first is very suitable.</p>
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\sqrt{x}	<p>the square root of x.</p>
------------	---

$\sqrt[3]{x}$	<p>the cube root of x.</p>
---------------	---

$\sqrt[5]{x}$	<p>the fifth root of x.</p>
---------------	--

$e^{\pi i} = -1$	<p>e to the (power of) π, i equals minus one. exponential to the power of π, i equals minus one.</p>
------------------	--

$h(2x, 3y)$	h of two x (comma) three y .
$h(x, y)$	h of x, y

$\prod_{k=1}^n A_k = \left(\prod_{k=1}^{n-1} A_k \right) \times A_n$	The product of A, k for k from one to n is equal to the product of A, k for k from one to n minus one times A, n .
---	--

$(n + 1)! = n! \cdot (n + 1), n = 0, 1, 2, \dots$	n plus one all factorial equals n factorial times n plus one, where n equals zero, one, two, ..., and so on.
---	--

$A = \{x \in R \mid x \leq p\}, \quad A' = \{x \in R \mid x \leq q\}.$	A equals the set of all x in R such that x is less or equal to p .
--	--

$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}.$	The binomial formula a plus b to the power of n is equal to the sum from k from zero to n of C, k, n (the binomial coefficient n over k) times a to the power of k times b to the power of n minus k .
--	---

$\binom{n}{k} = \frac{n!}{k! (n - k)!}$	C, k, n (the binomial coefficient n over k) equals n factorial over k factorial times n minus k factorial. (the binomial coefficient) n over k
---	--

$\langle f, g \rangle = \int_a^b f(x) g(x) dx$	The inner product of f and g equals the integral from a to b of f of x times g of x dx .
--	--

$\left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}.$	b over a all to the power of n equals b to the power of n over a to the power of n .
---	--

$\frac{a^n}{a^m} = a^{n-m}$	a to the power of n over a to the power of m equals a to the power of n minus m .
-----------------------------	---

$q = \sup M.$	q equals the sup of M .
---------------	-----------------------------

$\sum_{k=1}^n (x_k - x_{k-1}) = x_n - x_0.$	The sum for k from one to n of x_k minus x_{k-1} equals x_n minus x_0 .
---	---

$\bigcap_{n=1}^{\infty} [a_n, b_n] \neq \emptyset.$	The intersection of the closed intervals a_n, b_n for n from one to infinity is nonempty.
---	---

$\frac{a - p^n}{(p + 1)^n - p^n}.$	a minus p to the power of n all over p plus one to the power of n minus p to the power of n .
------------------------------------	---

$1 + \prod_{k=1}^n p_k$	One plus the product of p, k , for k from one to n . One plus the product, for k from one to n , of p, k .
-------------------------	---

$\left\ \frac{e^{xA} - I}{x} - A \right\ \leq \frac{e^{\ xA\ } - 1 - \ xA\ }{ x } = \left(\frac{e^{ x \ A\ } - 1}{ x } - \ A\ \right) \rightarrow 0$	The norm of exponential x, A minus I over x minus A is less or equal to exponential of the norm of x, A minus one minus the norm of x, A over the absolute value of x which is equal to exponential of the sum of the absolute value of a, k to power of p all to power of one over p , times absolute value of x times the norm of
--	---

	A minus one over the absolute value of x minus the norm of A which tends to zero.
--	---

$\det(A) = \lambda_1 \lambda_2 \dots \lambda_n = \prod_{\lambda_i \in Sp(A)} \lambda_i$	The determinant of A equals the product of lambda,i for i from one to n which is equal to the product of lambda,i, where lambda,i belongs to S,P (the spectre) of A.
---	--

$a > 1 \iff a^r > 1$	a is strictly larger than one if and only if a to the power r is strictly larger than one.
----------------------	--

$\sqrt[n]{a}$	The n-th root of a.
$\sqrt[5]{a}$	The fifth root of a.

$\left(\frac{1}{p^n}\right) < \frac{1}{a}$	One over p to the power of n is strictly less than one over a.
--	--

$x_1 + y_i$	x one plus y, i
-------------	-----------------

R_{ij}	R, i j capital R subscript i j capital R lower i j (capital) R (subscript) i j; (capital) R lower i j
----------	--

M_{ij}^k	(capital) M upper k lower i j; (capital) M superscript k subscript i j
------------	---

$\sum_{i=0}^n a_i x^i$	sum of a i x to the i for i from nought [= zero] to n; sum over i (ranging) from zero to n of a i (times) x to the i.
------------------------	--

$\prod_{m=1}^{\infty} b_m$	product of b_m for m from one to the infinity; product over m (ranging) from one to the infinity of b_m
----------------------------	--

$\sum_{i=0}^n \binom{n}{i} x^i y^{n-i}$	sum of n over i x to the i y to the n minus i for i from nought [= zero] to n .
---	---

$(x - y)^{3m}$	x minus y in brackets to the (power of) three m x minus y , all to the (power of) three m .
----------------	--

$\left \sum a_k b_k \right \leq \left(\sum a_k ^p \right)^{1/p} \left(\sum b_k ^q \right)^{1/q}$	The absolute value of the sum of $a_k b_k$ is less or equal to the sum of the absolute value of a_k to power of p all to power of one over p , times the sum of the absolute value of b_k to power of q all to power of one over q .
--	--

$D(E) = \{x \mid \ x\ \leq 1\}$,	D of E is equal to the set of all x such that the norm of x is less or equal to one.
------------------------------------	--

$\frac{1}{p} + \frac{1}{q} = 1$	One over p plus one over q equals one.
---------------------------------	--

$e^A = I_n + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \dots + \frac{A^n}{n!} + \dots$ $= \sum_{k=0}^{\infty} \frac{A^k}{k!}.$	Exponential A equals I_n plus A plus A squared over two factorial plus A cubed over three factorial Plus ... plus A to the power of n over n factorial plus, and so on which is equal to the sum of A to the power of k over k factorial, for k from zero to infinity.
--	--

$\ a + b\ _p \leq \ a\ _p + \ b\ _p.$	The norm of a plus b , p is less or equal to the the norm of a , p plus the norm of b , p .
---------------------------------------	---

$\ f\ _p = \left(\int_a^b f(x) ^p dx \right)^{1/p} < \infty.$	The norm of f , p equals the integral from a to b of the absolute value of f of x to the power of p d , x all to the power of one over p , is finite.
---	---

$\sup_{t \in [a, b]} x_n(t) - x(t) \rightarrow 0$	The sup, where t belongs to the closed interval a, b , of the absolute value of x, n of t minus x of t tends to zero.
---	---

$\lim_{n \rightarrow \infty} \left\ \sum_1^n \alpha_i e_i \right\ = \sqrt{\sum \alpha_i ^2}$	The limit as n tends to the infinity of the norm of the sum for I from one to n of α, i, e, i which equals the square root of the sum of the absolute value (the modulus) of α, i squared.
---	--

$F^{-1}(C) = f^{-1}(C) \cup g^{-1}(C)$	Big f to the minus one of C equals f to the minus one of C union g to the minus one of C .
--	--

\emptyset	The empty set (= set with no elements).
-------------	---

$\overline{f^{-1}(B)} \subset f^{-1}(\bar{B}).$	f minus one of B bar is a proper subset of f minus one of B bar.
---	--

$\lim_{n \rightarrow \infty} f(x_n) \neq f(x).$	The limit, as n tends to the infinity, of f of x, n is different from f of x .
---	--

$\lim_{n \rightarrow \infty} f(x_n) = f(x)$	The limit of f of x, n as n tends to infinity equals f of x .
---	---

$ \rho(x, Y) - \rho(z, Y) \leq \rho(x, z)$	The absolute value of rho of x, Y minus rho of z, Y is less or equal to
---	---

	rho of x,z.
--	--------------------

$\frac{d^2 y}{dx^2}$	the second derivative of y by x; d squared y by d x squared
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$\frac{\partial f}{\partial x}$	the partial derivative of f by x (with respect to x); partial d f by d x
---------------------------------	--

$\frac{\partial^2 f}{\partial x^2}$	the second partial derivative of f by x (with respect to x) partial d squared f by d x squared
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∇f	nabla f; the gradient of f
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Δf	delta f
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$A \subset Y \subset X$	A is a subset of Y which is a subset of X.
-------------------------	--

$\sum_{k=0}^{\infty} \frac{A^k}{k!}$	We consider the infinite series: The sum for k from zero to infinity of A to the power k over k factorial.
--------------------------------------	--

$\sum_{n=1}^{\infty} \ x_n\ < \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{2} \left(\frac{1}{1 - \frac{1}{2}}\right) = 1,$	The sum for n from one to infinity of the norm of x _n is strictly less than the sum for n from one to infinity of one half to the power of n which is equal to one half times one over one minus one half which equals one.
--	--

$\ (I - T)^{-1}\ \leq \frac{1}{1 - \ T\ }.$	The norm of I minus T all to the minus one is less or equal to one over one minus the norm of T.
--	--

$\sup_{A \in \mathcal{A}} \ A\ < \infty.$	<p>The sup of the norm of A, where A belongs to \mathcal{A}, is finite.</p>
$\langle Tx, Y \rangle = \langle x, T^*Y \rangle \quad \forall x, y \in H.$	<p>The inner product of T x, Y equals the inner product of x, T star Y, for every x, y belong to H.</p>
$A^2 \geq \sum_{j=1}^n \int_0^1 f_j(x) ^2 dx = \sum_{j=1}^n 1 = n$	<p>A squared is greater than or equal to the sum for j from one to n of the integral from zero to one of the absolute value of f, j of x squared d x, and this equals the sum for j from one to n of one which is equal to n.</p>
$\dim(\mathcal{M}) \leq A^2.$	<p>The dimension of M is less (than) or equal to A squared. Dim of M is less or equal to A squared.</p>
$f(x) = e_x(f) = \int_0^1 f(y) \overline{G(x, y)} dy \quad \text{for all } f \in \mathcal{M}.$	<p>f of x equals e, x of f which is equal to the integral from zero to one of f of y G of x, y bar d y, for all f belongs to M.</p>
$\ f\ _\infty \leq A \ f\ _p \leq A \ f\ _2$	<p>The norm of f, infinity, is less or equal to A times the norm of f, p which is less or equal to A times the norm of f, two.</p>
$E_n = \{x : \sup_{A \in \mathcal{A}} \ Ax\ \leq n\} = \bigcap_{A \in \mathcal{A}} \{x : \ Ax\ \leq n\}$	<p>E, n is equal to the set of all x such that sup of the norm of A, x, where x belongs to A is less or equal to n which equals the intersection of the sets of all x such that the norm of A, x is less or equal to n, where A belongs to \mathcal{A}</p>
$\sum_{n=1}^{\infty} \sup_{x \in E^c} f_n(x) \leq \sum_{n=1}^{\infty} M_n < \infty$	<p>The sum for n from one to infinity of sup of the absolute value of f, n of x, where x belongs to E, c is less or equal to the sum for n from one to infinity of M, n which is finite.</p>

$\left\ \sum_{n=1}^N c_n f_n \right\ _{\infty}^2 \leq B^2 \sum_{n=1}^N c_n ^2 \leq B^2 c ^2$	<p>The norm of the sum for n from one to N of c,n,f,n infinity squared is less or equal to B squared times the sum for n from one to N of the absolute value of c,n squared which is less or equal to B squared times the absolute value of c squared.</p>
---	--

$\ S - S_n\ _{\infty} \rightarrow 0 \text{ as } n \rightarrow \infty.$	<p>The norm of S minus S,n in the infinity tends to zero as n tends to infinity.</p>
--	--

$\int f(x) dx$	<p>integral of f of x d x</p>
$\int_a^b t^2 dt$	<p>integral from a to b of t squared d t</p>
$\iint_S h(x, y) dx dy$	<p>double integral over S of h of x y d x d y.</p>

$\ f\ = \left(\int_X f ^p d\mu \right)^{1/p}$	<p>The norm of f equals the integral over X of the absolute value of f to the power of p d,mu all to the power of one over p.</p>
--	---

$ gf = gf \text{ and } \left(\frac{ g }{\ g\ _q} \right)^q = \left(\frac{ f }{\ f\ _p} \right)^p \text{ a.e.}$	<p>The absolute value of g,f equals g,f and the absolute value of g over the norm of g,q to the power of q equals the absolute value of f over the norm of f,p to the power of p, about every.</p>
---	---

$A \in M_n(\mathbb{K})$	<p>A belongs to M,n of K. The matrix A belongs to M,n of K.</p>
-------------------------	---

$e^{-i\theta}$	<p>Exponential minus i, theta.</p>
----------------	---

$A = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$	<p>A is a squared matrix of order two defined by A equals cosine theta, sine theta, minus sine theta and cosine theta.</p>
---	--

$p_A(x) = \det(A - xI)$	<p>The characteristic polynomial of A : P,A of x is equal to the determinat of A minus x, I.</p>
-------------------------	--

x_0 **x zero; x nought**

$A^{-1} = \frac{1}{\det(A)} (\text{Com}(A))^t$	The inverse of A (A to the minus one) equals one over the determinant of A times the comatrix of a transpose.
--	---

$p_{AB}(\lambda) = p_{BA}(\lambda)$	P,A,B of lambda is equal to P,B,A of lambda, wehere p is characteristic polynomial.
-------------------------------------	---

$E_\lambda = \{x \in \mathbb{R}^n ; Ax = \lambda x\}$ $= \ker(A - \lambda I).$	The eigenspace associated with lambda equals the set of all x belongs to \mathbb{R},n such that $Ax = \lambda x$, which is equal to the kernel of A minus lambda I.
--	--

$f : \mathbb{P}_n[x] \longrightarrow \mathbb{P}_n[x]$ $p \longmapsto f(p) = p'$	f is an Endomorphism defined on the vector space \mathbb{P},n of x by f of p equals p prime.
---	--

$f^2 + 3f + 4id_E = 0$	f squared plus three f plus four times the identity mapping of E (plus four i,d,E) equals zero.
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<ul style="list-style-type: none"> ◆) $\forall x \in E : \ x\ \geq 0, \text{ et } \ x\ = 0 \Leftrightarrow x = 0$ ◆) $\forall \lambda \in \mathbb{K}, \forall x \in E : \ \lambda x\ = \lambda \cdot \ x\$ ◆) $\forall x, y \in E : \ x + y\ \leq \ x\ + \ y\ .$ 	For every x in E: the norm of x is positive or zero and the norm of x equals zero if and only if x equals zero For every lambda in K and for every x in E: the norm of lambda x equals the absolute value (the modulus) of lambda times the norm of x. For every x,y in E: the norm of x plus y is less or equal to the norm of x plus the norm of y.
--	---

$\ x\ _1 = \sum_{i=1}^n x_i , \quad \ x\ _2 = \left(\sum_{i=1}^n x_i ^2 \right)^{\frac{1}{2}},$ $\ x\ _\infty = \max_{1 \leq i \leq n} x_i .$	<p>Let x be an element from a vector space. The norm of x, one equals the sum for i from one to n of the absolute value of x_i.</p> <p>The square root of the norm of x, two equals the sum for i from one to n of x_i squared (of the modulus of x_i squared)</p> <p>The norm of x infinity is equal to the max for i from one to n of the absolute value of x_i (of the modulus of x_i).</p>
---	---

$\ A\ _1 = \max_j \sum_{i=1}^n a_{ij} , \quad \ A\ _\infty = \max_i \sum_{j=1}^n a_{ij} $	<p>The norm of the matrix A, one equals the max over j of the sum for i from one to n of the absolute value of a_{ij}.</p> <p>The norm of the matrix A infinity equals the max over i of the sum for j from one to n of the absolute value of a_{ij}.</p>
---	---

$\ Ax\ \leq \ A\ \ x\ ; \quad \forall A \in M_n(\mathbb{K}), \quad \forall x \in \mathbb{K}^n.$	<p>The norm of Ax is less or equal to the norm of A times the norm of x, for all A belongs to M_n of \mathbb{K} and for all x belongs to \mathbb{K}^n.</p>
---	--

$\diamond) \langle x, x \rangle \geq 0 \text{ et } \langle x, x \rangle = 0 \iff x = 0$ $\diamond) \langle x, y \rangle = \langle y, x \rangle \quad \forall x, y \in E$ $\diamond) \langle \lambda x, y \rangle = \lambda \langle x, y \rangle \quad \forall x, y \in E \text{ et } \forall \lambda \in \mathbb{R}$ $\diamond) \langle x, y + z \rangle = \langle x, y \rangle + \langle x, z \rangle \quad \forall x, y, z \in E$	<p>The scalar product of x, x is positive or zero and the scalar product of x, x equals zero if and only if x equals zero.</p> <p>The scalar product of x, y equals the scalar product of y, x for every x and y in E.</p> <p>The scalar product of $\lambda x, y$ equals λ times the scalar product of x, y for every x, y in E and λ in \mathbb{R}.</p> <p>The scalar product of x and $y + z$ equals the sum of the scalar product of x, y and x, z for every x, y, z in E.</p>
---	---

	plus z equals the scalar product of x,y plus the scalar product of y,z for every x,y,z in E.
--	--

$\langle x, y \rangle = \sum_{i=1}^n x_i y_i$	The inner product of x and y is equal to the sum, for i from one to n, of x _i (times) y _i .
---	---

$p_A(x) = \det(A - xI)$ $= \det((A - xI)^t)$ $= \det(A^t - xI)$ $= p_{A^t}(x).$	<p>The characteristic polynomial of A: p_A of x is equal to the determinant of A minus xI.</p> <p>Equals ...</p> <p>Equals ...</p> <p>Equals p_A transpose of x.</p>
---	--

$f : \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}$ $(x, y) \longmapsto x^t A y$	f is a mapping from R, n times R, n to R, defined by f of x,y equals x transpose A, y.
--	--

$\Delta f = 0$	the Laplace equation
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$\Delta f = \lambda f$	the Helmholtz equation
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$\Delta g = \frac{\partial g}{\partial t}$	the heat equation
--	-------------------

$\Delta g = \frac{\partial^2 g}{\partial t^2}$	the wave equation
--	-------------------

$\lim_{t \rightarrow 0} \frac{e^{At} - I}{t} = A.$	The limit as t tends to zero of exponential A,t minus I over t equals A.
--	--

$\begin{aligned} \lambda \langle x, y \rangle &= \langle \lambda x, y \rangle = \langle Ax, y \rangle \\ &= \langle x, A^t y \rangle = \langle x, Ay \rangle \\ &= \langle x, \beta y \rangle = \beta \langle x, y \rangle \end{aligned}$	<p>Lambda times the inner product of x,y equals the inner product of lambda x,y and this equals the inner product of A x,y</p> <p>which equals beta times the inner product of x,y.</p>
---	---

$(A^t A)^t = A^t (A^t)^t = A^t A.$	<p>A transpose A, all transpose equals A transpose A transpose, transpose which equals A transpose, A.</p>
------------------------------------	--

$\alpha_0 A^m + \alpha_1 A^{m-1} + \dots + \alpha_m I$	<p>Alpha zero times A to the m plus alpha one times A to the m minus one plus ... plus alpha m time I.</p>
--	--

$M = \underbrace{\frac{1}{2} (M - M^t)}_A + \underbrace{\frac{1}{2} (M + M^t)}_B$	<p>The matrix M is always written as the sum of two matrices A and B, where A equals M minus M transpose over two and B equals M plus M transpose over two.</p>
---	---

$M_{n \times n}(\mathbb{R}) = S_n(\mathbb{R}) \oplus A_n(\mathbb{R})$	<p>M,n of R is equal to the direct sum of S,n of R and A,n of R.</p>
---	--

$(B^t = -B)$	<p>B transpose equals (is equal to) minus B.</p>
--------------	--

$\begin{aligned} \lambda \langle x, x \rangle &= \langle \lambda x, x \rangle \\ &= \langle Ax, x \rangle = (Ax)^t \bar{x} \\ &= x^t A^t \bar{x} = x^t \left((\overline{A})^t \right)^t \bar{x} \\ &= x^t \overline{A} \bar{x} = x^t \overline{Ax} \\ &= \langle x, Ax \rangle = \langle x, \lambda x \rangle = \bar{\lambda} \langle x, x \rangle \end{aligned}$	<p>Lambda times the inner product of x,x equals the inner product of lambda x,x which equals the inner product of A x,x and this equals A x transpose, x bar</p> <p>which equals lambda bar times the inner product of x,x.</p>
--	---

$A^{-1} = A^*$	<p>A to the minus one equals A star. The inverse of A is equal to A star.</p>
----------------	---

	The inverse of A equals A star.
--	---------------------------------

$$A^{-1} = \frac{-1}{c_0} \sum_{k=1}^n c_k A^{k-1}$$

A minus one equals minus one over c, zero times the sum for k from one to n of c, k, A to the power k minus one.

$A^k = P B^k P^{-1}$	A to the power of k equals P times B to the power of k times P minus one.
----------------------	---

$$A^t A = A A^t = I_n$$

$$A^t = A^{-1}$$

$$\|Ax\| = \|x\|; \forall x \in \mathbb{R}^n.$$

$$(Ax)^t (Ay) = x^t y; \forall x, y \in \mathbb{R}^n.$$

A transpose, A, equals A, A transpose which is equal to I, n.

A transpose equals A to the minus one.

The norm of A, x is equal to the norm of x, for all x belongs to R, n.

A x transpose A, y equals x transpose, y, for all x and y belong to R, n.

$ax^2 + bx + c$	a x squared plus b x plus c
-----------------	-----------------------------

$\sqrt{x} + \sqrt[3]{y}$	the square root of x plus the cube root of y
--------------------------	--

$\sqrt[n]{x + y}$	the n-th root of x plus y
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$\frac{a+b}{c-d}$	a plus b over c minus d
-------------------	-------------------------

$\binom{n}{m}$	(the binomial coefficient) n over m
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For any positive integer n ,

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors}}$$

where a is the **base** and n is the **exponent**.

6.3. Sets and spaces

\mathbb{N} The set of *natural numbers*

\mathbb{Z} The set of *integers*

\mathbb{R} The set of *real numbers*

\mathbb{C} The set of *complex numbers*

$|x|$ absolute value of a real or complex numbers x

$]a,b[$ an open interval, $[a,b]$ a closed interval

Notations & notions

- A mapping with **domain** X and **range** in Y

$$f : X \longrightarrow Y$$

- Element inclusion or in set, set inclusion, union and intersection :

$$\in \subset \cup \cap$$

- Vectors and matrices**

$$x = (x_1, x_2, \dots, x_n) \text{ a row vector in } \mathbb{R}^n \text{ or } \mathbb{C}^n \text{ with components } x_i$$

The **transpose** of x and the **adjoint** of x

$$x^t = (x_1, x_2, \dots, x_n)^t = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$
$$x^* = (\overline{x_1}, \overline{x_2}, \dots, \overline{x_n})^t$$

The **transpose**, **adjoint**, **inverse**, **determinant**, **condition number** and the **spectral radius** of the matrix A .

$$A^t \quad A^* \quad A^{-1} \quad \det(A) \quad \text{Cond}(A) \quad \rho(A)$$

Eigenvalues and Eigenvectors. Consider a square matrix A . A nonzero vector x is an **eigenvector** of the matrix with **eigenvalue** l if $Ax = lx$

We see that $l = 2$ is an **eigenvalue** of **multiplicity** 3.

$$\det \begin{pmatrix} 1 & 7 & 4 \\ 0 & -1 & 2 \end{pmatrix} \text{ is not possible}$$

Part 7. On certain mathematical subfields

1) Algebra ['ældʒɪbrə]

- Algebraic Curves
- Algebraic Equations
- Algebraic Geometry
- Algebraic Identities
- Algebraic Invariants
- Algebraic Operations
- Algebraic Properties
- Coding Theory ['kəʊdɪŋ 'θɪəri]
- Cyclotomy ['saɪk'lɒtəmɪ]
- Elliptic Curves [ɪ'lɪptɪk kɜ:v]
- Field Theory
- General Algebra
- Group Theory
- Homological Algebra
- Linear Algebra
- Named Algebras
- Noncommutative Algebra
- Number Theory
- Polynomials [ˌpɒlɪ'nəʊmɪəlz]
- Products
- Quadratic Forms
- Quaternions and Cliffo...
- Rate Problems
- Ring Theory
- Scalar Algebra
- Sums [sʌmz]
- Valuation Theory
- Vector Algebra
- Wavelets ['weɪvlɪts]

[ˌvæljʊ'eɪʃən 'θɪəri] [ˈnʌmbə 'θɪəri] [neɪmd 'ældʒɪbrəs]

[kwə'tɜ:nɪənz]

2) Applied Mathematics [ə'plɑɪd ,mæθə'mætɪks]

- Business ['bɪznɪs]
- Complex Systems ['kɒmpleks 'sɪstəmz]
- Control Theory [kən'trəʊl 'θɪəri]
- Data Visualization ['deɪtə ,vɪzjʊəlaɪ'zeɪʃən]
- Dynamical Systems [daɪ'næmɪkəl 'sɪstəmz]
- Engineering [ˌendʒɪ'nɪəriŋ]
- Ergodic Theory [ˈɜ:gədɪk 'θɪəri]
- Game Theory [geɪm 'θɪəri]
- Information Theory [ˌɪnfə'meɪʃən 'θɪəri]
- Inverse Problems [ˈɪnvɜ:s 'prɒbləmz]
- Numerical Methods [nju:'merɪkəl 'meθəds]
- Optimization [ˌɒptɪmaɪ'zeɪʃən]
- Population Dynamics [ˌpɒpjʊ'leɪʃən daɪ'næmɪks]
- Signal Processing ['sɪgnl 'prəʊsesɪŋ]

3) Calculus and Analysis ['kælkjʊləs ænd, ə'næləsɪs]

- Calculus
- Calculus of Variations
- Catastrophe Theory [kə'tæstrəfɪ]
- Complex Analysis
- Differential Equations
- Differential Forms
- Differential Geometry
- Dynamical Systems
- Fixed Points
- Functional Analysis
- Functions
- General Analysis
- General Calculus
- Generalized Functions
- Harmonic Analysis
- Inequalities
- Integral Transforms
- Inversion Formulas
- Manifolds ['mænɪfəʊldz]
- Measure Theory
- Metrics
- Norms
- Operator Theory
- Polynomials
- Roots
- Series
- Singularities
- Special Functions [ˌsɪŋgjʊ'ləɪtɪz]

4) Discrete Mathematics [dɪs'kri:t ,mæθə'mætɪks]

- Cellular Automata ['seljʊlə 'ɔ:təmətə]
- Coding Theory ['kəʊdɪŋ 'θɪəri]
- Combinatorics

- Computational Systems [ˌkɒmpjʊ'teɪʃənəl 'sɪstəmz]
- Computer Science [kəm'pju:tər 'saɪəns]
- Division Problems [dɪ'vɪʒən 'prɒbləmz]
- Experimental Mathematics [ɪk,sperɪ'mentl ,mæθə'mæɪɪks]
- Finite Groups ['faɪnaɪt gru:p]
- General Discrete Mathematics ['dʒenərəl dɪs'kri:t ,mæθə'mæɪɪks]
- Graph Theory [grɑ:f 'θɪəri]
- Information Theory [ˌɪnfə'meɪʃən 'θɪəri]
- Packing Problems ['pækɪŋ 'prɒbləmz]
- Point Lattices [pɔɪnt 'lætɪsɪz]
- Recurrence Equations [rɪ'kʌrəns ɪ'kwɛɪʒən]
- Umbral Calculus [ʌm'brəl 'kælkjʊləs]

5) Foundations of Mathematics [faʊn'deɪʃənz əv ,mæθə'mæɪɪks]

- Axioms ['æksɪəm]
- Category Theory ['kætiɡəri 'θɪəri]
- Logic ['lɒdʒɪk]
- Mathematical Problems [ˌmæθə'mæɪɪkəl 'prɒbləmz]
- Point-Set Topology [pɔɪnt-set 'θɪəri]
- Set Theory [set 'θɪəri]
- Theorem Proving ['θɪərəm pru:vɪŋ]

6) Geometry [dʒɪ'ɒmɪtri]

- | | | |
|--------------------------|-----------------------------|-----------------------|
| ■ Algebraic Geometry | ■ Ergodic Theory | ■ Plane Geometry |
| ■ Combinatorial Geometry | ■ General Geometry | ■ Points |
| ■ Computational Geometry | ■ Geometric Construction | ■ Projective Geometry |
| ■ Continuity Principle | ■ Geometric Duality | ■ Rigidity |
| ■ Coordinate Geometry | ■ Geometric Inequalities | ■ Sangaku Problems |
| ■ Curves | ■ Inversive Geometry | ■ Solid Geometry |
| ■ Differential Geometry | ■ Line Geometry | ■ Surfaces |
| ■ Dissection [dɪ'sekʃən] | ■ Multidimensional Geometry | ■ Symmetry |
| ■ Distance | ■ Noncommutative Geometry | ■ Transformations |
| ■ Division Problems | ■ Non-Euclidean Geometry | ■ Trigonometry |

7) History and Terminology [ˈhɪstəri ænd, ,tɜ:mɪ'nɒlədʒɪ]

- Biography [baɪ'ɒɡrəfi]
- Contests [kən'tests]
- Disciplinary Terminology [ˈdɪsɪplɪnəri ,tɜ:mɪ'nɒlədʒɪ]
- History ['hɪstəri]
- Mathematica Code [ˌmæθə'mæɪɪkæ kəʊd]
- Mathematica Commands [ˌmæθə'mæɪɪkæ kə'mɑ:ndz]
- Mathematical Problems [ˌmæθə'mæɪɪkəl 'prɒbləmz]
- Mnemonics [nɪ'mɒnɪks]
- Notation [nəʊ'teɪʃən]
- Prizes [praɪzɪz]
- Terminology [ˌtɜ:mɪ'nɒlədʒɪ]

8) Number Theory ['nʌmbər 'θɪəri]

- Algebraic Number Theory
- Arithmetic
- Automorphic Forms
- Binary Sequences
- Class Numbers
- Congruences
- Constants
- Continued Fractions
- Diophantine Equations
- Divisors [dɪ'vaɪzəʳz]
- Elliptic Curves
- Ergodic Theory
- General Number Theory
- Generating Functions
- Integer Relations
- Integers
- Irrational Numbers
- Normal Numbers
- Numbers
- Number Theoretic Funct...
- Parity ['pærɪtɪ]
- Prime Numbers
- p-adic Numbers
- Rational Approximation
- Rational Numbers
- Real Numbers
- Reciprocity Theorems
- Rounding
- Sequences
- Special Numbers
- Transcendental Numbers

9) Probability and Statistics [ˌprɒbə'bɪlɪtɪ ænd, stə'tɪstɪks]

- Bayesian Analysis
- Descriptive Statistics
- Error Analysis
- Estimators ['estɪmeɪtəʳz]
- Markov Processes
- Moments ['mɒmənts]
- Multivariate Statistics
- Nonparametric Statistics
- Probability
- Random Numbers
- Random Walks ['rændəm]
- Rank Statistics
- Regression [rɪ'greʃən]
- Runs [rʌnz]
- Statistical Asymptotic...
- Statistical Distributions
- Statistical Indices ['ɪndɪsiːz]
- Statistical Plots [plɒts]
- Statistical Tests
- Time-Series Analysis
- Trials ['traɪəlz]

[dɪs'krɪptɪv stə'tɪstɪks] ['mɑːkɒv 'prəʊses ɪːz]

10) Recreational Mathematics [ˌrekrɪ'eɪʃənəl ˌmæθə'mætɪks]

- Cryptograms ['krɪptəʊgræmz]
- Dissection [dɪ'sekʃən]
- Folding ['fɒlɪdɪŋ]
- Games [geɪmz]
- Illusions [ɪ'luːʒən]
- Magic Figures ['mædʒɪk 'fɪgəʳ]
- Mathematical Art [ˌmæθə'mætɪkəl ɑːt]
- Mathematical Humor [ˌmæθə'mætɪkəl 'hjuːmər]
- Mathematical Records [ˌmæθə'mætɪkəl rɪ'kɔːdz]
- Mathematics in the Arts [ˌmæθə'mætɪks ɪn ðɪ, ɑːt]
- Number Guessing ['nʌmbəʳ ɡesɪŋ]
- Numerology [ˌnjuːmə'rɒlədʒɪ]
- Puzzles ['pʌzlz]
- Sports [spɔːts]

Numerical Methods

- Approximation Theory
- Differential Equation Solving
- Finite Differences
- Linear Systems
- Numerical Integration
- Numerical Summation
- Root-Finding

Interpolation

- Aitken Interpolation
- B-Spline
- Berlekamp-Massey Algorithm
- Bernstein-Bézier Curve
- Bézier Curve
- Bézier Spline
- Bezigon
- Bicubic Spline
- Bulirsch-Stoer Algorithm
- C-Determinant
- Cardinal Function
- Chebyshev Approximatio...
- Cubic Spline
- Gauss's Interpolation...
- Hermite's Interpolatin...
- Internal Knot
- Interpolant
- Interpolation
- Lagrange Interpolating...
- Lagrange Interpolation
- Lagrangian Coefficient
- Lebesgue Constants
- Moving Average
- Moving Median
- Muller's Method
- Neville's Algorithm
- Newton's Divided Diffe...
- NURBS Curve
- NURBS Surface
- Richardson Extrapolation
- Spline
- Thiele's Interpolation...
- Thin Plate Spline

Fixed Points

- Cosine Constant
- Cosine Fixed Point Constant
- Cosine Superposition Constant
- Dottie Number
- Elliptic Fixed Point
- Fixed Point
- Fixed Point Node
- Fixed Point Star
- Fixed Point Theorem
- Heteroclinic Point
- Hillam's Theorem
- Hopf Bifurcation
- Hyperbolic Fixed Point
- Improper Node
- Iterated Cosine Constant
- Lefschetz Fixed Point formula
- Lefschetz Trace Formula
- Map Cycle
- Map Fixed Point
- Newton's Method
- Parabolic Fixed Point
- Poincaré-Birkhoff Fixed point Theorem
- Schauder Fixed Point Theorem
- Spiral Point
- Stable Improper Node
- Stable Node
- Stable Spiral Point
- Stable Star
- Unstable Improper Node
- Unstable Node
- Unstable Spiral Point
- Unstable Star

Operator Theory

- Adomian Polynomial
- Anticommutative
- Anticommutator
- Antilaplacian
- Antilinear
- Antisymmetric
- Antiunitary
- Closable Operator
- Closed Operator
- Conjugate Transpose
- Cyclic Operator
- Cyclic Vector
- Generalized Hilbert Algebra
- Hermitian Operator
- Identity Operator
- Involution
- Left Hilbert Algebra
- Linear Operator
- Modular Automorphism
- Modular Automorphism Group
- Modular Conjugation
- Modular Involution
- Modular Operator
- Operand
- Operator
- Operator Extension
- Operator Spectrum
- Operator Theory
- Perron-Frobenius Operator
- Preclosed Operator
- Right Hilbert Algebra
- Scattering Operator
- Scattering Theory
- Separating Vector
- Shift-Invariant Operator
- Shift Operator
- Tomita-Takesaki Theory
- Unimodular Hilbert Algebra
- Unitary
- Wave Operator
- Weierstrass Operator

Measure Theory

- A-Integrable
- Absolutely Continuous
- Almost Surely
- Banach Measure
- Borel Hierarchy
- Cantor Set
- Carathéodory Derivative
- Carathéodory Extension
- Carathéodory Measure
- French Metro Metric
- Frullani's Integral
- G_delta Set
- Gauss Measure
- Haar Integral
- Hausdorff Measure
- Hausdorff Paradox
- Helson-Szegö Measure
- HK Integral
- Measure Theory
- Measure Zero
- Minkowski Measure
- Monotonic Function
- Mutual Energy
- Open Cluster
- Outer Measure
- Partition Function
- Perron Integral

- Completeness Property
- Complex Measure
- Correlation Dimension
- Countable Additivity
- Countable Monotonicity
- Countable Subadditivity
- Darboux Integral
- Darboux-Stieltjes Integral
- Denjoy Integral
- Denjoy-Saks-Young Theorem
- Egoroff's Theorem
- Egorov's Theorem
- Energy
- Essential Supremum
- Euclidean Metric
- Euler Integral
- F_sigma Set
- Fatou's Lemma
- Fatou's Theorems
- Finite Additivity
- Finite Monotonicity
- Finite Subadditivity
- Fréchet Derivative
- Hopf's Theorem
- Jordan Measure
- Jordan Measure Decomposition
- KMS Condition
- Kubo-Martin-Schwinger condition
- Lebesgue Covering Dimension
- Lebesgue Decomposition
- Lebesgue Integrable
- Lebesgue Integral
- Lebesgue Measure
- Lebesgue-Stieltjes Integral
- Lebesgue Sum
- Liouville Measure
- Lusin's Theorem
- Mahler Measure
- Measurable Function
- Measurable Set
- Measurable Rectangle
- Measurable Space
- Measure
- Measure Algebra
- Measure Space
- Pincherle Derivative
- Pointwise Convergence
- Polar Representation
- Positive Measure
- Premeasure
- Product Measure
- Radon Measure
- Radon-Nikodym Derivative
- Radon-Nikodym Theorem
- Random-Cluster Model
- Real Measure
- Riemann Sum
- Riesz Representation Theorem
- Sard's Theorem
- Scrawny Cantor Set
- Set Function
- Singular Measure
- Square Integrable
- Stieltjes Integral
- Titchmarsh Theorem
- Total Variation
- Lebesgue's Dominated Convergence Theorem

Differential Forms

- Alternating Bilinear Form
- Angle Bracket
- Bilinear Form
- Bra
- Differential
- Differential Form
- Differential k-Form
- Dirac Notation
- Eigenform
- Exact Differential
- Exact Form
- Exterior Derivative
- Exterior Power
- Hasse-Minkowski Theorem
- Hodge Star
- Inexact Differential
- Kähler Form
- Ket
- Kodaira Embedding Theorem
- Lagrange Bracket
- Legendre Transformation
- Multilinear Form
- One-Form
- Pfaffian Form
- Poincaré's Lemma
- Projection Operator
- Rectifiable Current
- Stokes' Theorem
- Sylvester's Signature
- Symmetric Inner Product
- Two-Form
- Volume Form
- Wedge Product
- Zero-Form

Part 8. English Grammar series

English Grammar series 1

Ex01. Make the past simple.

He _____ (tell) me that he lived in Toronto. We _____ (lend) John £200. She _____ (drink) too much coffee yesterday. The children _____ (sleep) in the car. I _____ (forget) to buy some milk. They _____ (speak) French to the waitress. He _____ (feel) terrible after eating the meal. I _____ (know) the answer yesterday.

Ex02. Put in the correct preposition: England is famous.....its rainy weather. He isn't afraid..... anything. English cheese is very different..... french cheese. Who is James married.....? Lucy is extremely good.....languages. Are you pleased.....your new house? That bike is similaryours. I'm very excitedbuying a new computer. What is your town famous.....? My niece is afraiddogs. Julie is very different.....her sister. She's very excited.....the party. I've been married.....my husband for 10 years. Unfortunately, I'm very bad.....music. Luke is very pleasedhis exam results. He isn't really interested.....getting married. I'm very proud.....my daughter, she worked very hard.

Ex03. Reported Statements: Present Simple.

1. "I live in New York" She said _____
2. "He works in a bank" She told me _____
3. "Julie doesn't like going out much" She said _____
4. "I don't have a computer" She said _____
5. "They never arrive on time" She said _____
6. "We often meet friends in London at the weekend" He told me _____
7. "She doesn't have enough time to do everything" She said _____
8. "They often go on holiday in July" She said _____

Ex04. Put the verb into the correct first conditional form:

1. If I _____ (go) out tonight, I _____ (go) to the cinema.
2. If you _____ (get) back late, I _____ (be) angry.
3. If we _____ (not / see) each other tomorrow, we _____ (see) each other next week.
4. If he _____ (come), I _____ (be) surprised.
5. If we _____ (wait) here, we _____ (be) late.
6. If we _____ (go) on holiday this summer, we _____ (go) to Spain.
7. If the weather _____ (not / improve), we _____ (not / have) a picnic.
8. If I _____ (not / go) to bed early, I _____ (be) tired tomorrow.
9. If we _____ (eat) all this cake, we _____ (feel) sick.

10. If you _____ (not / want) to go out, I _____ (cook) dinner at home.

Ex05. Second Conditionals - put the verb into the correct tense:

1. If I _____ (be) you, I _____ (get) a new job.
2. If he _____ (be) younger, he _____ (travel) more.
3. If we _____ (not / be) friends, I _____ (be) angry with you.
4. If I _____ (have) enough money, I _____ (buy) a big house.
5. If she _____ (not / be) always so late, she _____ (be) promoted.
6. If we _____ (win) the lottery, we _____ (travel) the world.
7. If you _____ (have) a better job, we _____ (be) able to buy a new car
8. If I _____ (speak) perfect English, I _____ (have) a good job.
9. If we _____ (live) in Mexico, I _____ (speak) Spanish.
10. If she _____ (pass) the exam, she _____ (be)
11. She _____ (be) happier if she _____ (have) more friends.
12. We _____ (buy) a house if we _____ (decide) to stay here.
13. They _____ (have) more money if they _____ (not / buy) so many clothes
14. We _____ (come) to dinner if we _____ (have) time.
15. She _____ (call) him if she _____ (know) his number.
16. They _____ (go) to Spain on holiday if they _____ (like) hot weather.
17. She _____ (pass) the exam if she _____ (study) more.
18. I _____ (marry) someone famous if I _____ (be) a movie star.
19. We never _____ (be) late again if we _____ (buy) a new car.
20. You _____ (lose) weight if you _____ (eat) less.

Ex06. Make future simple questions:

1. _____ (they / come) tomorrow?
2. When _____ (you / get) back?
3. If you lose your job, what _____ (you / do)?
4. In your opinion, _____ (she / be) a good teacher?
5. What time _____ (the sun / set) today?
6. _____ (she / get) the job, do you think?
7. _____ (David / be) at home this evening?
8. What

_____ (the weather / be) like tomorrow? 9. There's someone at the door,
_____ (you / get) it? 10. How _____ (he / get) here?

Ex07. Make the present continuous ('yes / no' or 'wh' questions).

1. (how long / you / stay in Paris?)

2. (you / drink / tea ?)

3. (where / you / stay?)

4. (why / you / watch TV now?)

5. (she / work in a bank?)

6. (what / he / do?)

7. (why / she / call her friend now?)

8. (I / lose weight?)

9. (we / work tomorrow?)

10. (when / you / arrive?)

Ex08. Choose the present simple or the present continuous.

1. Julie _____ (read) in the garden.

2. What _____ (we / have) for dinner tonight?

3. She _____ (have) two daughters.

4. I _____ (stay) in Spain for two weeks this summer.

5. He often _____ (come) over for dinner.

6. The class _____ (begin) at nine every day.

7. What _____ (you / eat) at the moment?

8. What _____ (Susie / do) tomorrow?

9. I _____ (not / work) on Sundays.

10. She _____ (not / study) now, she _____ (watch) TV.

11. How often _____ (you / go) to restaurants?

12. I _____ (not / go) on holiday this summer.

13. I'm sorry, I _____ (not / understand).

14. She _____ (work) as a waitress for a month.

15. She _____ (take) a salsa dancing class every Tuesday.

16. It _____ (be) cold here in winter.

17. Take your umbrella, it _____ (rain).

18. This cake _____ (taste) delicious.

19. The bag _____ (belong) to Jack.

20. When _____ (you / arrive) tonight?

Ex09. Change these sentences from active to passive:

1. People speak Portuguese in Brazil.

2. The Government is planning a new road near my house.

3. My grandfather built this house in 1943.

4. Picasso was painting Guernica at that time.

5. The cleaner has cleaned the office.

6. He had written three books before 1867.

7. John will tell you later. _____

8. By this time tomorrow we will have signed the deal.

9. Somebody should do the work.

10. The traffic might have delayed Jimmy. _____

11. Everybody loves Mr Brown.

12. They are building a new stadium near the station.

13. The wolf ate the princess.

14. At six o'clock someone was telling a story.

15. Somebody has drunk all the milk!

16. I had cleaned all the windows before the storm.

17. A workman will repair the computer tomorrow.

18. By next year the students will have studied the passive.

19. James might cook dinner.

20. Somebody must have taken my wallet.

English Grammar series 2

Ex10. Make the past simple.

1. She _____ (bring) some chocolates to the party.
2. I _____ (hear) a new song on the radio.
3. I _____ (read) three books last week.
4. They _____ (speak) French to the waitress.
5. He _____ (understand) during the class, but now he doesn't understand.
6. I _____ (forget) to buy some milk.
7. She _____ (have) a baby in June.
8. You _____ (lose) your keys last week.
9. They _____ (swim) 500m.
10. I _____ (give) my mother a CD for Christmas.
11. At the age of 23, she _____ (become) a doctor.
12. I _____ (know) the answer yesterday.
13. He _____ (tell) me that he lived in Toronto.
14. We _____ (lend) John £200.
15. She _____ (drink) too much coffee yesterday.
16. The children _____ (sleep) in the car.
17. He _____ (keep) his promise.
18. I _____ (choose) the steak for dinner.
19. The film _____ (begin) late.
20. We _____ (fly) to Sydney.
21. They _____ (drive) to Beijing.
22. He _____ (teach) English at **the** University.
23. I _____ (send) you an e-mail earlier.
24. We _____ (leave) the house at 7 a.m.
25. He _____ (feel) terrible after eating the prawns.

Ex11. Change these sentences from active to passive.

1. Somebody cleans the office every day. **Ans.** *The office is cleaned every day*
2. Somebody sends emails. _____
3. Somebody cuts the grass. _____

4. Somebody prefers chocolate.

5. Somebody often steals cars.

6. Somebody plays loud

music. _____

7. Somebody speaks English here.

8. Somebody loves the London parks.

9. Somebody wants staff. _____

10. Somebody writes articles.

11. Somebody loves Julie. _____

12. Somebody reads a lot of books.

13. Somebody cooks dinner everyday.

14. Somebody delivers milk in the mornings.

15. Somebody buys flowers for the flat.

16. Somebody washes the cars every week.

17. Somebody writes a report every Friday.

18. Somebody fixes the roads.

19. Somebody builds new

houses every year. _____

20. Somebody sells vegetables in the market.

Ex12. Tag Questions with the Present Simple (be careful: 'I am' → 'aren't I' but 'I'm not' → 'am I'). **Add the tag question:**

1. She's from a small town in China, _____ ?

2. They aren't on their way already, _____ ?

3. We're late again, _____ ?

4. I'm not the person with the tickets, _____ ?

5. Julie isn't an accountant, _____ ?
6. The weather is really bad today, _____ ?
7. He's very handsome, _____ ?
8. They aren't in Mumbai at the moment, _____ ?
9. You aren't from Brazil, _____ ?
10. John's a very good student, _____ ?
11. I like chocolate very much, _____ ?
12. She doesn't work in a hotel, _____ ?
13. They need some new clothes, _____ ?
14. We live in a tiny flat, _____ ?
15. She studies very hard every night, _____ ?
16. David and Julie don't take Chinese classes, _____ ?
17. I often come home late, _____ ?
18. You don't like spicy food, _____ ?
19. She doesn't cook very often, _____ ?
20. We don't watch much TV, _____ ?

Ex12. Combine the following two sentences.

1. We ate the fruit. I bought the fruit.

2. They called a lawyer. The lawyer lived nearby.

3. I sent an email to my brother. My brother lives in Australia.

4. The customer liked the waitress. The waitress was very friendly.

5. We broke the computer. The computer belonged to my father.

6. I dropped a glass. The glass was new.

7. She loves books. The books have happy endings.

8. They live in a city. The city is in the north of England.

9. The man is in the garden. The man is wearing a blue jumper.

10. The girl works in a bank. The girl is from India.

11. My sister has three children. My sister lives in Australia.

12. The waiter was rude. The waiter was wearing a blue shirt.

13. The money is in the kitchen. The money belongs to John.

14. The table got broken. The table was my grandmother's.

15. The television was stolen. The television was bought 20 years ago.

16. The fruit is on the table. The fruit isn't fresh.

17. The food was delicious. David cooked the food.

English Grammar series 3

Ex01. Choose 'however', 'although' or 'despite':

1. _____ the rain, we still went to the park.
2. _____ it was raining, we still went to the park.
3. It was raining. _____, we still went to the park.
4. John bought the watch, _____ the fact that it was expensive.
5. John bought the watch. _____, it was expensive.
6. _____ it was expensive, John bought the watch.
7. I finished the homework. It, _____, wasn't easy.
8. I finished the homework, _____ it wasn't easy.
9. _____ the fact that it wasn't easy, I finished the homework.
10. She went for a long walk, _____ being cold.
11. _____ she was cold, she went for a long walk.
12. She was cold. She went for a long walk, _____.
13. The restaurant has a good reputation. _____, the food was terrible.
14. _____ the restaurant's good reputation, the food was terrible.
15. _____ the restaurant has a good reputation, the food was terrible.

Ex 02. Choose the correct word or phrase in brackets to fill the space.

1. (because / because of) We stayed inside _____ the storm.
2. (since / because of) I wanted to stay longer _____ I was really enjoying the party.
3. (as / due to) Amanda stayed at home _____ her illness.
4. (due to / as) Her lateness was _____ a terrible traffic jam.
5. (since / owing to) _____ flights are cheaper in the winter, we decided to travel then.
6. (as / because of) _____ she hated cats, she wasn't happy when her husband bought three.
7. (owing to / as) John didn't go to work _____ his illness.
8. (because / due to) _____ Lucy was very tired, she went to bed early. 9. (because / owing to) _____ his late night, John missed his train.

10. (for / owing to) Lucy was very unhappy, _____ she missed James. 11. (as / due to) _____ the terrible weather, we decided not to walk home.
12. (as / owing to) I was very happy with my present, _____ it was exactly what I wanted.
13. (due to / since) Keiko ordered her meal without meat, _____ she is a vegetarian.
14. (because of / as) I didn't want to leave _____ I was having a great time.
15. (owing to / since) Luca bought the shoes _____ they were perfect. 16. (because of / because) We were late for the plane _____ the traffic.
17. (for / as) _____ it was really cold, I put on my gloves and my hat. 18. (due to / because) She couldn't come _____ she had to work.
19. (owing to / because) _____ its high price, we didn't rent the flat. 20. (because of / since) _____ his great cooking, we love going to dinner at Taka's house.

Ex 03. Choose the correct form (adjective or adverb).

- 1) John held the plate _____. (careful / carefully)
- 2) Julia is a _____ person. (careful / carefully)
- 3) I ran _____ to the station. (quick / quickly)
- 4) The journey was _____. (quick / quickly)
- 5) You look _____. Didn't you sleep well? (tired / tiredly)
- 6) The baby rubbed her eyes _____. (tired / tiredly)
- 7) She sang _____. (happy / happily)
- 8) You sound _____. (happy / happily)
- 9) I speak English _____. (well / good)
- 10) Her English is _____. (well / good)
- 11) She cooks _____. (terrible / terribly)
- 12) He is a _____ cook. (terrible / terribly)
- 13) The music was _____. (beautiful / beautifully)
- 14) She plays the piano _____. (beautiful / beautifully)
- 15) That was a _____ answer. (clever / cleverly)
- 16) She answered _____. (clever / cleverly)
- 17) Your flat seems _____ today. (tidy / tidily)

- 18) He put the dishes away _____. (tidy / tidily)
 19) He spoke _____. (warm / warmly)
 20) She is a very _____ person. (warm / warmly)

Ex 04. Put the verb into the correct form:

1. I don't fancy _____ (go) out tonight.
2. She avoided _____ (tell) him about her plans.
3. I would like _____ (come) to the party with you.
4. He enjoys _____ (have) a bath in the evening.
5. She kept _____ (talk) during the film.
6. I am learning _____ (speak) English.
7. Do you mind _____ (give) me a hand?
8. She helped me _____ (carry) my suitcases.
9. I've finished _____ (cook). Come and eat!
10. He decided _____ (study) Biology.
11. I dislike _____ (wait).
12. He asked _____ (come) with us.
13. I promise _____ (help) you tomorrow.
14. We discussed _____ (go) to the cinema, but in the end we stayed at home.
15. She agreed _____ (bring) the pudding.
16. I don't recommend _____ (take) the bus, it takes forever!
17. We hope _____ (visit) Amsterdam next month.
18. She suggested _____ (go) to the museum.
19. They plan _____ (start) college in the autumn.
20. I don't want _____ (leave) yet.

Ex 05. Put in the correct preposition (**at, in, on**, or no preposition):

1. There was a loud noise which woke us up ____ midnight.
2. Do you usually eat chocolate eggs ____ Easter?
3. What are you doing ____ the weekend?
4. ____ last week, I worked until 9pm ____ every night.
5. My father always reads the paper ____ breakfast time.
6. She plays tennis ____ Fridays.
7. The trees here are really beautiful ____ the spring.
8. I'll see you ____ Tuesday afternoon, then.
9. Shakespeare died ____ 1616.
10. She studies ____ every day.
11. John is going to buy the presents ____ today.
12. In my hometown the shops open early ____ the morning.
13. She met her husband ____ 1998.
14. The party is ____ next Saturday.
15. We are meeting ____ Friday morning.
16. I often get sleepy ____ the afternoon.
17. His daughter was born ____ the 24th of August.
18. Mobile phones became popular ____ the nineties.
19. The meeting will take place ____ this afternoon.
20. Luckily the weather was perfect ____ her wedding day.

English Grammar series 4

Ex 01. Make the comparative form. If it's possible, use 'er'. If not, use 'more'.

1. Dogs are _____ (intelligent) than rabbits.
2. Lucy is _____ (old) than Ellie.
3. Russia is far _____ (large) than the UK.
4. My Latin class is _____ (boring) than my English class.
5. In the UK, the streets are generally _____ (narrow) than the streets in the USA.
6. London is _____ (busy) than Glasgow.
7. Julie is _____ (quiet) than her sister.
8. Amanda is _____ (ambitious) than her classmates.
9. My garden is a lot _____ (colourful) than this park.
10. His house is a bit _____ (comfortable) than a hotel.

Ex 02. Change the direct questions into indirect questions. Use 'can you tell me'.

1) Where does she play tennis?

2) Does he live in Paris?

3) Is she hungry?

4) What is this?

5) Do they work in Canada?

6) When do John and Luke meet?

7) Is he a lawyer?

8) When is the party?

9) Do they often go out?

10) What does he do at the weekend?

11) Are the children on holiday this week?

12) Who is she?

13) Why do you like travelling so much?

14) Does Lizzie like ice cream?

15) Are they from Chile?

16) Where is the station?

17) Where do you study Chinese?

18) Where is the nearest supermarket?

19) Do you drink coffee?

20) Is Richard always late?

Ex 03. If it's possible, make a sentence with 'would + infinitive'. If it's not possible, use 'used to + infinitive':

1. I / have short hair when I was a teenager.

2. We / go to the same little café for lunch every day when I was a student.

3. She / love playing badminton before she hurt her shoulder.

4. He / walk along the beach every evening before bed.

5. I / always lose when I played chess with my father.

6. She / be able to dance very well.

7. My grandfather / drink a cup of coffee after dinner every night.

8. Luke / not have a car.

9. We / live in Brazil.

10. My family / often go to the countryside for the weekend when I was young.

Ex 04. Put the corresponding prepositions of Place

1. The wine is _____ the bottle. 2. Pass me the dictionary, it's _____ the bookshelf. 3. Jennifer is _____ work. 4. Berlin is _____ Germany. 5. You have something _____ your face. 6. Turn left _____ the traffic lights. 7. She was listening to classical music _____ the radio. 8. He has a house _____ the river. 9. The answer is _____ the bottom of the page. 10. Julie will be _____ the plane now. 11. There are a lot of magnets _____ the fridge. 12. She lives _____ London. 13. John is _____ a taxi. He's coming. 14. I'll meet you _____ the airport. 15. She stood _____ the window and looked out. 16. The cat is _____ the house somewhere. 17. Why you calling so late? I'm already _____ bed. 18. I waited for Lucy _____ the station. 19. There was a picture of flowers _____ her T-shirt.

20. She has a house _____ Japan.

Ex 05. Change this direct speech into reported speech:

1. "He works in a bank" She said

2. "We went out last night" She told me

3. "I'm coming!" She said

4. "I was waiting for the bus when he arrived" She told me

5. "I'd never been there before" She said

6. "I didn't go to the party" She told me

7. "Lucy'll come later" She said

8. "He hasn't eaten breakfast" She told me

9. "I can help you tomorrow" She said

10. "You should go to bed early" She told me

11. "I don't like chocolate" She told me

12. "I won't see you tomorrow" She said

13. "She's living in Paris for a few months" She said

14. "I visited my parents at the weekend" She told me

15. "She hasn't eaten sushi before" She said

16. "I hadn't travelled by underground before I came to London" She said

17. "They would help if they could" She said

18. "I'll do the washing-up later" She told me

19. "He could read when he was three" She said

20. "I was sleeping when Julie called" She said

English Grammar series 5

Ex 01. Make the future perfect. Choose positive, negative or question.

1. (I / leave by six) _____
2. (you / finish the report by the deadline?)

3. (when / we / do everything?)

4. (she / finish her exams by ten, so we can go out for dinner)

5. (you / read the book before the next class)

6. (she / not / finish work by seven)

7. (when / you / complete the work?)

8. (they / arrive by dinnertime)

9. (we / be in London for three years next week)

10. (she / get home by lunchtime?) _____

Ex 02. Put in 'can' / 'can't' / 'could' / 'couldn't'. If none is possible, use 'be able to' in the correct tense:

1. _____ you swim when you were 10?
2. We _____ get to the meeting on time yesterday because the train was delayed by one hour.
3. He _____ arrive at the party on time, even after missing the train, so he was very pleased.
4. He's amazing, he _____ speak 5 languages including Chinese.
5. I _____ drive a car until I was 34, then I moved to the countryside so I had to learn.
6. I looked everywhere for my glasses but I _____ find them anywhere.
7. I searched for your house for ages, luckily I _____ find it in the end.
8. She's 7 years old but she _____ read yet – her parents are getting her extra lessons.
9. I read the book three times but I _____ understand it.
10. James _____ speak Japanese when he lived in Japan, but he's forgotten most of it now.

Ex 03. Put in the correct preposition:

1. He's swimming _____ the river.
2. Where's Julie? She's _____ school.
3. The plant is _____ the table.
4. There is a spider _____ the bath.

5. Please put those apples _____ the bowl.
6. Frank is _____ holiday for three weeks.
7. There are two pockets _____ this bag.
8. I read the story _____ the newspaper.
9. The cat is sitting _____ the chair.
10. Lucy was standing _____ the bus stop.
11. I'll meet you _____ the cinema.
12. She hung a picture _____ the wall.
13. John is _____ the garden.
14. There's nothing _____ TV tonight.
15. I stayed _____ home all weekend.
16. When I called Lucy, she was _____ the bus.
17. There was a spider _____ the ceiling.
18. Unfortunately, Mrs Brown is _____ hospital.
19. Don't sit _____ the table, sit _____ a chair.
20. There are four cushions _____ the sofa.

1. England is famous _____ its rainy weather.
2. I'm very proud _____ my daughter, she worked very hard.
3. He isn't really interested _____ getting married.
4. Luke is very pleased _____ his exam results.
5. Unfortunately, I'm very bad _____ music.
6. I've been married _____ my husband for 10 years.
7. She's very excited _____ the party.
8. Julie is very different _____ her sister.
9. My niece is afraid _____ dogs.
10. A ball gown is similar _____ an evening dress.
11. What is your town famous _____?
12. It's great you got that job - you should be proud _____ yourself.
13. I'm very excited _____ buying a new computer.
14. That bike is similar _____ yours.
15. She is interested _____ jazz.
16. Are you pleased _____ your new house?
17. Lucy is extremely good _____ languages.
18. Who is James married _____?
19. English cheese is very different _____ French cheese.
20. He isn't afraid _____ anything.

English Grammar series 6

Ex 01. Put in 'mustn't' or 'don't / doesn't have to':

1. We have a lot of work tomorrow. You _____ be late.
2. You _____ tell anyone what I just told you. It's a secret.
3. The museum is free. You _____ pay to get in.
4. Children _____ tell lies. It's very naughty.
5. John's a millionaire. He _____ go to work.
6. I _____ do my washing, because my mother does it for me.
7. We _____ rush. We've got plenty of time.
8. You _____ smoke inside the school.
9. You can borrow my new dress but you _____ get it dirty.
10. We _____ miss the train, it's the last one tonight.
11. She _____ do this work today, because she can do it tomorrow.
12. I _____ clean the floor today because I cleaned it yesterday.
13. We _____ forget to lock all the doors before we leave.
14. We _____ stay in a hotel in London, we can stay with my brother.
15. I _____ spend too much money today. I've only got a little left.
16. They _____ get up early today, because it's Sunday.
17. I _____ eat too much cake, or I'll get fat!
18. We _____ be late for the exam.
19. You _____ tidy up now. I'll do it later.
20. He _____ cook tonight because he's going to a restaurant

Ex 02. Change the **direct questions** into **indirect questions**. Use 'Do you know'.

1) Did she go out last night?

2) Where did she meet her brother?

3) How was the film?

4) Was David the first to arrive?

5) Did Lucy work at home yesterday?

6) What was the problem?

7) Who did we see at the party?

8) Did Zac call his mum yesterday?

9) Were they at the beach?

10) Where was the class?

11) Why did they arrive so late?

12) Was she at home yesterday?

13) How did she do it?

14) Were they in the garden?

15) Did they arrive late?

16) Did John finish the report?

17) Were we late for the meeting?

18) What did they do at the weekend?

19) Why was she so early?

20) Where was Julie yesterday afternoon?

Ex 03. Make the **comparative form**. If it's possible, use 'er'. If not, use 'more'.

1. Dogs are _____ (intelligent) than rabbits.

2. Lucy is _____ (old) than Ellie.

3. Russia is far _____ (large) than the UK.

4. My Latin class is _____ (boring) than my English

class. 5. In the UK, the streets are generally _____
(narrow) than the streets in the USA.

6. London is _____ (busy) than Glasgow.

7. Julie is _____ (quiet) than her sister.

8. Amanda is _____ (ambitious) than her classmates.

9. My garden is a lot _____ (colourful) than this park.

10. His house is a bit _____ (comfortable) than a hotel.

Ex 04. Choose **a little** / **little** / **a few** / **few**:

1. I have _____ water left. There's enough to share.

2. I have _____ good friends. I'm not lonely.

3. He has _____ education. He can't read or write, and he can hardly count.

4. There are _____ people she really trusts. It's a bit sad.

5. We've got _____ time at the weekend. Would you like to meet?
6. Julie gave us _____ apples from her garden. Shall we share them?
7. She has _____ self-confidence. She has a lot of trouble talking to new people.
8. There are _____ women politicians in the UK. Many people think there should be more.
9. It's a great pity, but the hospital has _____ medicine. They can't help many people.
10. I've got _____ cakes to give away. Would you like one?
11. There's _____ milk left in the fridge. It should be enough for our coffee.
12. _____ children from this school go on to university, unfortunately.
13. Do you need information on English grammar? I have _____ books on the topic if you would like to borrow them.
14. She's lucky. She has _____ problems.
15. London has _____ sunshine in the winter. That's why so many British people go on holiday to sunny places!
16. There's _____ spaghetti left in the cupboard. Shall we eat it tonight?
17. There are _____ programmes on television that I want to watch. I prefer to download a film or read a book.
18. He has _____ free time. He hardly ever even manages to call his mother!
19. Unfortunately, I have _____ problems at the moment.
20. Are you thirsty? There's _____ juice left in this bottle, if you'd like it

Ex 05. Fill the gap with **the** if it's necessary.

1. Everest is _____ highest mountain in the world.
2. Who is _____ oldest person in your family?
3. This dress was _____ cheapest.
4. Which language do you think is _____ easiest to learn?
5. This book is _____ most serious one on the topic.
6. I think that one over there is _____ strongest horse.
7. This film is _____ shortest.
8. She's _____ fastest runner in her school.
9. That suitcase is _____ lightest.
10. Out of all the cities in Europe, London is _____ biggest.

Ex 06. Fill the gap with '**some**' or '**no article**'.

1. Can you buy _____ pasta? [I'm thinking of the amount we need for tonight.]
2. We need _____ mushrooms [I'm not thinking about the amount].
3. John drinks _____ coffee every morning [coffee, not tea].
4. Add _____ water to the soup if it's too thick [a certain amount of water].
5. I really want _____ tea – could you get me a cup?
6. We could have _____ rice for dinner [rice, not pasta].
7. I ate _____ bread and two eggs for lunch [I'm thinking about the amount].
8. She bought _____ new furniture [a certain amount of furniture].
9. Did you get _____ carrots? [I'm not thinking about the amount.]
10. I'd like _____ tea, please! [Tea, not juice or coffee.]

Ex 07. Put in the correct preposition (**at**, **in**, **on**, or no preposition):

1. There was a loud noise which woke us up _____ midnight.

2. Do you usually eat chocolate eggs ____ Easter?
3. What are you doing ____ the weekend?
4. ____ last week, I worked until 9pm ____ every night.
5. My father always reads the paper ____ breakfast time.
6. She plays tennis ____ Fridays.
7. The trees here are really beautiful ____ the spring.
8. I'll see you ____ Tuesday afternoon, then.
9. Shakespeare died ____ 1616.
10. She studies ____ every day.
11. John is going to buy the presents ____ today.
12. In my hometown the shops open early ____ the morning.
13. She met her husband ____ 1998.
14. The party is ____ next Saturday. 15. We are meeting ____ Friday morning.
16. I often get sleepy ____ the afternoon.
17. His daughter was born ____ the 24th of August.
18. Mobile phones became popular ____ the nineties.
19. The meeting will take place ____ this afternoon.
20. Luckily the weather was perfect ____ her wedding day.

Ex 08. Change the **direct questions** into **reported questions**:

1. Where is the post office? She asked me

2. Why is Julie sad? She asked me

3. What's for dinner? She asked me

4. Who is the woman in the red dress? She asked me

5. How is your grandmother? She asked me

6. When is the party? She asked me

7. How much is the rent on your flat? She asked me

8. Where are the glasses? She asked me

9. How is the weather in Chicago? She asked me

10. Who is the Prime Minister of Canada? She asked me

11. Where do you usually go swimming? She asked me

12. What does Luke do at the weekend? She asked me

13. Where do your parents live? She asked me

14. Who do you go running with? She asked me

15. When does Lucy get up? She asked me

16. How much TV do you watch? She asked me

17. How many books do they own? She asked me

18. Where does John work? She asked me

19. What do the children study on Fridays? She asked me

20. Why do you study English? She asked me

Ex 09. Change the **direct speech** into **reported speech**:

1. "Please help me carry this" She asked me

2. "Please come early" She

3. "Please buy some milk" She

4. "Could you please open the window?" She

5. "Could you bring the book tonight?" She

6. "Can you help me with my homework, please?" She

7. "Would you bring me a cup of coffee, please?" She

8. "Would you mind passing the salt?" She

9. "Would you mind lending me a pencil?" She

10. "I was wondering if you could possibly tell me the time." She

11. "Do your homework!" She told me

12. "Go to bed!" She

13. "Don't be late!" She

14. "Don't smoke!" She

15. "Tidy your room!" She

16. "Wait here!" She

17. "Don't do that!" She

18. "Eat your dinner!" She

19. "Don't make a mess!" She

20. "Do the washing-up!" She

Ex 10. Make the positive **future continuous**: **At three o'clock tomorrow...**

1. I _____ (work) in my office.
2. You _____ (lie) on the beach.
3. He _____ (wait) for the train.
4. She _____ (shop) in New York.
5. It _____ (rain).
6. We _____ (get) ready to go out.
7. They _____ (meet) their parents.
8. He _____ (study) in the library.
9. She _____ (exercise) at the gym.
10. I _____ (sleep).

Ex 11. Make future continuous 'yes / no' questions: **When the boss comes,**

1. _____ (I / sit) here?
2. _____ (John / us) the computer?
3. _____ (Jane and Luke / discuss) the new project?
4. _____ (we / work) hard?
5. _____ (you / talk) on the telephone?
6. _____ (she / send) an email?
7. _____ (they / have) a meeting?
8. _____ (he / eat) lunch?
9. _____ (you / type)?
10. _____ (he / make) coffee?

Make 'wh' future continuous questions: **At 8pm,**

11. (where / I / wait?) At 8pm, _____
12. (what / you / do?) _____
13. (why / he / study?) _____
14. (how / she / travel?) _____
15. (who / they / meet?) _____
16. (where / we / eat?) _____
17. (what / you / watch?) _____
18. (why / he / drive?) _____
19. (what / she / cook?) _____

20. (why / they / sleep?) _____

Ex 12. Change these **direct questions** into **reported speech**:

1. "Where is he?" She asked me

2. "What are you doing?" She asked me

3. "Why did you go out last night?" She asked me

4. "Who was that beautiful woman?" She asked me

5. "How is your mother?" She asked me

6. "What are you going to do at the weekend?" She asked me

7. "Where will you live after graduation?" She asked me

8. "What were you doing when I saw you?" She asked me

9. "How was the journey?" She asked me

10. "How often do you go to the cinema?" She asked me

11. "Do you live in London?" She asked me

12. "Did he arrive on time?" She asked me

13. "Have you been to Paris?" She asked me

14. "Can you help me?" She asked me

15. "Are you working tonight?" She asked me

16. "Will you come later?" She asked me

17. "Do you like coffee?" She asked me

18. "Is this the road to the station?" She asked me

19. "Did you do your homework?" She asked me

20. "Have you studied reported speech before?" She asked me

Ex 13. Conditional exercise (first / second / third conditionals)

1. (First conditional) If we _____ (not / work) harder, we
_____ (not pass) the exam.

2. (Third conditional) If the students _____ (not be) late for the exam, they _____ (pass).
3. (Third conditional) If the weather _____ (not be) so cold, we _____ (go) to the beach.
4. (Second conditional) If she _____ (have) her laptop with her, she _____ (email) me.
5. (First conditional) If she _____ (not go) to the meeting, I _____ (not go) either.
6. (Third conditional) If the baby _____ (sleep) better last night, I _____ (not be) so tired.
7. (First conditional) If the teacher _____ (give) us lots of homework this weekend, I _____ (not be) happy.
8. (Second conditional) If Lucy _____ (have) enough time, she _____ (travel) more.
9. (First conditional) If the children _____ (not eat) soon, they _____ (be) grumpy.
10. (First conditional) If I _____ (not go) to bed soon, I _____ (be) tired in the morning.
11. (Second conditional) If I _____ (want) a new car, I _____ (buy) one.
12. (Second conditional) If José _____ (not speak) good French, he _____ (not move) to Paris.
13. (First conditional) If John _____ (drink) too much coffee, he _____ (get) ill.
14. (Third conditional) If we _____ (tidy) our flat, we _____ (not lose) our keys.
15. (Third conditional) If Luke _____ (not send) flowers to his mother, she _____ (not be) happy.
16. (Second conditional) If the children _____ (be) in bed, I _____ (be able to) have a bath.
17. (Second conditional) If you _____ (not be) so stubborn, we _____ (not have) so many arguments!
18. (Third conditional) If Julie _____ (not go) to Sweden, she _____ (go) to Germany.
19. (First conditional) If she _____ (go) to the library, she _____ (study) more.
20. (Third conditional) If we _____ (not have) an argument, we _____ (not be) late.
21. (Second conditional) If you _____ (arrive) early, it _____ (be) less stressful.
22. (Third conditional) If I _____ (not go) to the party, I _____ (not meet) Amanda.
23. (Second conditional) If Julie _____ (like) chocolate, I _____ (give) her some.
24. (Second conditional) If Luke _____ (live) in the UK, I _____ (see) him more often.

25. (Third conditional) If the children _____ (not eat) all that chocolate, they _____ (feel) sick.
26. (First conditional) If they _____ (not / arrive) soon, we _____ (be) late.
27. (Third conditional) If she _____ (study) Mandarin, she _____ (go) to Beijing.
28. (Second conditional) If we _____ (not be) so tired, we _____ (go) out.
29. (First conditional) If you _____ (buy) the present, I _____ (wrap) it up.
30. (First conditional) If Lucy _____ (not quit) her job soon, she _____ (go) crazy.

Ex 14. Choose ‘**however**’, ‘**although**’ or ‘**despite**’:

1. _____ the rain, we still went to the park.
2. _____ it was raining, we still went to the park.
3. It was raining. _____, we still went to the park.
4. John bought the watch, _____ the fact that it was expensive.
5. John bought the watch. _____, it was expensive.
6. _____ it was expensive, John bought the watch.
7. I finished the homework. It, _____, wasn't easy.
8. I finished the homework, _____ it wasn't easy.
9. _____ the fact that it wasn't easy, I finished the homework.
10. She went for a long walk, _____ being cold.
11. _____ she was cold, she went for a long walk.
12. She was cold. She went for a long walk, _____.
13. The restaurant has a good reputation. _____, the food was terrible.
14. _____ the restaurant's good reputation, the food was terrible.
15. _____ the restaurant has a good reputation, the food was terrible

Ex 15. Choose the correct word or phrase in brackets to fill the space.

1. (because / because of) We stayed inside _____ the storm.
2. (since / because of) I wanted to stay longer _____ I was really enjoying the party.
3. (as / due to) Amanda stayed at home _____ her illness.
4. (due to / as) Her lateness was _____ a terrible traffic jam.
5. (since / owing to) _____ flights are cheaper in the winter, we decided to travel then.
6. (as / because of) _____ she hated cats, she wasn't happy when her husband bought three.
7. (owing to / as) John didn't go to work _____ his illness.
8. (because / due to) _____ Lucy was very tired, she went to bed early.
9. (because / owing to) _____ his late night, John missed his train.

10. (for / owing to) Lucy was very unhappy, _____ she missed James. 11. (as / due to) _____ the terrible weather, we decided not to walk home.
12. (as / owing to) I was very happy with my present, _____ it was exactly what I wanted.
13. (due to / since) Keiko ordered her meal without meat, _____ she is a vegetarian.
14. (because of / as) I didn't want to leave _____ I was having a great time.
15. (owing to / since) Luca bought the shoes _____ they were perfect. 16. (because of / because) We were late for the plane _____ the traffic.
17. (for / as) _____ it was really cold, I put on my gloves and my hat. 18. (due to / because) She couldn't come _____ she had to work.
19. (owning to / because) _____ its high price, we didn't rent the flat. 20. (because of / since) _____ his great cooking, we love going to dinner at Taka's house.

Ex 16*. Choose the correct **phrasal verb**: go on / pick up / come back / come up with / go back / find out / come out / go out / point out / grow up / set up / turn out / get out / come in(to) / take on.

1. Can you _____ (think of an idea) a better idea? 2. She _____ (showed / mentioned) that the shops would already be closed.
3. I wish I hadn't _____ (become responsible for) so much work!
4. I _____ (went to an event) for dinner with my husband last night.
5. He _____ (entered a place where the speaker is) the kitchen and made some tea.
6. Where did you _____ (become an adult)?
7. I'd love to _____ (arrange / create) my own business.
8. I really want to _____ (leave a building) of this office and go for a walk.
9. As I arrived, he _____ (appeared from a place) of the door.
10. She _____ (got something from a place) some dinner on the way home.
11. Could you _____ (get information) what time we need to arrive?
12. I thought the conference was going to be boring but it _____ (in the end we discovered) to be quite useful.

13. What time did you _____ (return to a place where the speaker is) yesterday?
14. She _____ (appeared from a place) of the café and put on her gloves.
15. A performance _____ (is happening) at the moment.
16. He _____ (left a car) of the car.
17. He _____ (went to an event) a lot at the weekend, so he's tired today.
18. Can we _____ (arrange / create) a meeting next week?
19. Would anybody like to _____ (become responsible for) this new client?
20. He _____ (returned to a place where the speaker is) before I left.
21. It's lovely watching my children _____ (become adults).
22. She _____ (returned to a place where the speaker is not) to school.
23. He _____ (showed / mention) the stars to the children.
24. He _____ (returned to a place where the speaker is not) to Poland last year.
25. He _____ (thought of an idea) a solution.
26. Please _____ (enter a place where the speaker is)!
27. At the end of the film, it _____ (in the end we discovered) that John was a good guy.
28. Could you _____ (get someone from a place) Lucy later?
29. We need to _____ (get information) how much it costs.
30. What _____ ('s happening)?

Ex 17.** Choose the correct **phrasal verb**: give up / make up / end up / get back / look up / figure out / sit down / get up / take out / come on / go down / show up / take off / work out / stand up.

1. She _____ (arrived somewhere again) to London last week.
2. David _____ (removed clothes or jewellery) his gloves and put them in his pocket.
3. John _____ (changed from lying or sitting to standing – not casual) and left the room without a word.
4. What time did John _____ (arrive somewhere again) yesterday?

5. Please _____ (I am encouraging the person to go faster or try harder)! We're already miles behind the others.
6. She _____ (raised her eyes) from her laptop and smiled.
7. She _____ (stopped having or doing) coffee last year but was so grumpy that she started drinking it again.
8. You should _____ (remove clothes or jewellery) your hat inside.
9. People from other countries _____ (are the parts that form something) about a third of the population of London.
10. He didn't even _____ (raise his eyes) when she came in. So rude!
11. We _____ (are moving to a lower place) to the beach now. Would you like to come?
12. Lucy! _____ (change from lying or sitting to standing – more casual) quick! The teacher is coming!
13. The children _____ (changed from lying or sitting to standing – less casual) when the headmaster arrived.
14. She came into the room and _____ (changed from standing to sitting).
15. She _____ the bags _____ (removed from a container) of the car and put them in the hall.
16. _____ (I'm encouraging the person to go faster or try harder)! You're doing really well. Keep going!
17. She finally _____ (thought until she understood / planned – more UK) the answers to the maths homework.
18. They _____ (moved to a lower place) to the kitchen and made some tea.
19. Please _____ (change from standing to sitting). You're making me nervous!
20. She _____ (changed from lying or sitting to standing – more casual) slowly and picked up her bag.
21. After a long day, we _____ (finally did or were something, especially when you don't expect it) getting a pizza and falling asleep on the sofa.
22. I can't _____ (think about until I understand / plan – more UK) why Andrew is so upset.
23. We'll _____ (stop having or doing) chocolate after the holidays!
24. I can't _____ (think about until I understand / plan more USA) how to do this exercise.
25. I couldn't believe it! He didn't _____ (arrive at or come to an event / meeting - especially if there's something surprising) until 11pm!

26. They _____ (thought about until they understood / plan – more USA) that it must have been the toddler who put the milk in the oven.

27. She wanted to go travelling but she _____ (finally did or was something, especially when you don't expect it) working in a shop all summer.

28. I _____ the book _____ (removed from a container) of my bag and gave it to him.

29. I think coffee and cakes _____ (are the parts that form something) most of her diet!

30. We had a meeting yesterday but only a few people _____ (arrive at or come to an event / meeting - especially if there's something surprising).

Ex 18.** Choose the correct **phrasal verb**: come down (from) / go ahead / go up / look back (on) / wake up / carry out / take over / hold up / pull out (of) / turn around / take up / look down / put up / bring back / bring up.

1. Now that Amanda has quit, we really need someone to _____ (take control of) that part of the business.

2. If you want to get started on the report, please _____ (being to do something).

3. We were talking about the weather and then she _____ (started to talk about) the election.

4. This project _____ (is using a certain amount of space or time) far too much time.

5. The repairs were _____ (done and finished a task or activity) by a carpenter.

6. John is going to _____ (take control of) the project.

7. Why don't you _____ (begin to do something) and have dinner? I'll join you later.

8. She _____ (moved from a high place to a lower place) the attic with dust in her hair.

9. I _____ (changed from sleeping to being awake) in the middle of the night and I couldn't go back to sleep.

10. The scientists _____ (did and finished a task or activity) an experiment.

11. He felt something touch his leg so he _____ (moved his eyes down) and saw a cat.

12. _____ (change from sleeping to being awake)! We need to go out!

13. She often _____ (thought about something in the past) her time at university.

14. We used to call each other every week. Let's _____ that _____ (make something return) – it was really good.

15. She _____ her purse _____ (took something out of a container) her pocket.
16. Her phone call _____ (used a certain amount of space or time) the whole evening.
17. Let's _____ (fix something where it will be seen) a notice to tell people the class has been cancelled.
18. Please _____ (move from a high place to a lower place) that tree! It's not safe!
19. Her house will probably _____ (increase) in value.
20. The stock market _____ (has increased) recently.
21. She _____ (moved her eyes down) at her feet and mumbled an answer.
22. She _____ (fixed something where it will be seen) some photos of her family next to her bed.
23. Please _____ (hold something high up so people can see it) the sign so I can read it.
24. He likes to _____ (think about something in the past) his trip round Australia.
25. She _____ (held something high up so people can see it) the camera.
26. He _____ the conversation _____ (make something return) to the problem.
27. We walked to the lake, then _____ (moved to face the other direction) and came back.
28. Could you _____ (start to talk about) this problem at the meeting, please?
29. He _____ (moved to face the other direction) and left the room.
30. He reached into the drawer and _____ (took something out of a container) a notebook and pen.

Part 9. Final Exams from 2012 to 2016

In this part, we present all the tests, final exams and passing exams, which are programmed for Master students at University of 08 Mai 1945 Guelma, from the year 2012 to 2016 by the author.

Exercise 01 : Re-write the following words in ordinary English

- ['si:kwəns], ['vælju:], [tə'pɒlədʒɪ], ['ɒpəreɪtəʃ]
- ['fɔ:mjələ], [,ɛkspəʊ'nɛnʃəl], [,mæθə'mætɪks], [pru:f]
- [,ɪn'hæʊmə'dʒɪ:nɪəs], ['i:vən], [ɒd], ['kəʊsaɪn]
- ['sʌbgru:p], [,dʒenərəlaɪ'zeɪʃən], [saɪn], ['meɪʒəʃ].

Exercise 02 : Give one word for every symbol

ə	i:	ɪ	æ	e	ʌ	
ɔ:	ɒ	a:	u:	ʊ	ə:	
eɪ	eə	aɪ	ɔɪ	aʊ	ɪə	ʊə

Exercise 03 : Complete the following diagram:

rain ~~bike~~ warm boot
late leave five room
woke born bean phone

Exercise 04 : Give the **phonetic** of the following words and phrases

CHAPTER, Operation, Roots, Point, Mathematical induction, lower bound of a sequence, Infinite series, Rules for differentiation, MULTIPLE INTEGRALS, Power series, Orthogonal functions, Residue theorem, The rabbits raced right around the ring.

Exercise 05 : Translate the following sentences in English language.

- En particulier, si la série $\sum u_n$ converge, on obtient le théorème 2 à partir du critère de Cauchy en prenant $p = 0$.
- Soit E un espace vectoriel sur \mathbb{K} . Le produit scalaire $\langle \cdot, \cdot \rangle$ est une application définie par :
- De plus, on a d'après le lemme (2), on peut écrire :
- Ce qui achève la démonstration.

Exercise 06 : Complete the following sentences by using the correspondent mathematical notions:

1) \mathbb{Z} The set of

2) A mapping with X and in Y denoted by

$$f : X \longrightarrow Y$$

3) Consider a square matrix A . A nonzero vector x is an of the matrix with ℓ if

$$Ax = \ell x$$

4) A has the general form

$$p_n(x) = a_0 + a_1x + \dots + a_nx^n$$

5) Every of a convergent sequence converges to the same limit.

6) The with **(1,1)** and **3** has the equation

$$(x - 1)^2 + (y - 1)^2 = 9$$

7) **Theorem :** A nonempty set S of reals which is bounded above has the

8) Show that the series

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

is

9) Let \mathcal{R} be an equivalence relation on a set A . That means

\mathcal{R} (1) is, (2) is, (3) is

10) The interior of A is the union of all open sets in A

11) The closure of A is the intersection of all closed sets A

12) A vector v is a of the vectors x , y and z if it can be written as

$$v = \alpha x + \beta y + \gamma z ;$$

where α, β, γ are constants.

13)**Set**

A set of a *space* H satisfying

$$\langle e_i, e_j \rangle = \begin{cases} 1 ; i = j \\ 0 ; i \neq j \end{cases}$$

14) A subset E of a *vector space* V is called a of V if each *vector* $x \in V$ can be uniquely written in the form

$$x = \sum_{i=1}^n \alpha_i e_i ; e_i \in E$$

15) The $f * g$ of two functions f and g is given by

$$(f * g)(x) = \int f(x - y) g(y) dy$$

16) Let $\{ e_1, e_2, \dots \}$ be an orthonormal basis in a Hilbert space (H, \langle, \rangle) . Then every $x \in H$ can be written as a Fourier

$$x = \sum_{i \in I} \langle x, e_i \rangle e_i$$

The $\langle x, e_i \rangle$ are called the Fourier of x .

Exercise 07 : Give the conjugation of the verb ” **to go**” and complete the table

Present simple He goes	Present continuous He	Present perfect He	Present perfect continuous He
Past simple He	past continuous He	Past perfect He	Past perfect continuous He
Future simple He	Future continuous He	Future perfect He	Future perfect continuous He
Conditional present simple He	Conditional present continuous He	Conditional perfect simple He	Conditional perfect continuous He

Good luck

Bellavour. Djamel

Inner product space

In mathematics, a vector space or function space in which an operation for combining two vectors or functions (whose result is called an inner product) is defined and has certain properties. Such spaces, an essential tool of functional analysis and vector theory, allow analysis of classes of functions rather than individual functions. In mathematical analysis, an inner product space of particular importance is a *Hilbert* space, a generalization of ordinary space to an infinite number of dimensions.

A point in a *Hilbert* space can be represented as an infinite sequence of coordinates or as a vector with infinitely many components. The inner product of two such vectors is the sum of the products of corresponding coordinates. When such an inner product is zero, the vectors are said to be orthogonal. *Hilbert* spaces are an essential tool of mathematical physics.

Exercise 01 : (7.5 marks)

- 1) Give another title of the text. (1 mark).
- 2) Find a word or expression in the text which, in context, is similar in meaning to :

Farness, Series, unlimited, interior, boundless (2.5 marks)

- 3) Re-write the following words in ordinary English (4 marks)

- [ɪnɪ'kwɒlɪtɪ], [ɪ'nɪʃəl], [pɒlɪ'nəʊmɪəl], [ʌn'bændɪd]
- ['ældʒɪbrə], [ɪ'senʃəl], ['neglɪdʒəbl], [dɪfərənsɪ'eɪʃən]
- [nju:'merɪkəl], [dʒɪ'ɒmɪtrɪ], [prɒbə'bɪlɪtɪ], ['prɒpətɪ]
- [kəm'pli:t], [ju:'klɪdɪən], ['faɪnəɪt, dɪ'menʃən].

Exercise 02 (4 marks) : Give the phonetic of the following words and phrases

Dimension, operator, orthogonal projection, regular point, self-adjoint , spectrum, minimizing sequence, Countable, triangle inequality, uniform boundedness, Lax–Milgram theorem, successive approximations, extension, differential.

Exercise 03 (2 marks) : Translate the second paragraph of the text in French language.

Exercise 04 (2 marks) : Translate the following paragraph in English language.

Théorème : Pour qu'un espace métrique (E, d) soit complet, il faut et il suffit que, toute suite décroissante de boules fermées de rayons tendant vers zero admette une intersection non vide. (plus précisément, cette intersection est réduite à un seul élément).

Exercice 05 (3 marks) : Complete the following sentences by using the correspondent mathematical notions

1) **Series :** A series that signs, *i.e.*, of the form :

$$\sum (-1)^n a_n ; a_n \geq 0$$

2) Let P be a matrix of of a given symmetric matrix A and D a matrix of the corresponding Then, A can be written as :

$$A = PDP^{-1}$$

where D is a diagonal matrix.

3) Let A be a bounded linear operator on a *Hilbert* space, H . Then the value of A is given by :

$$|A| = \sqrt{A^*A}$$

where A^* is the of A .

4) **Set :**

A set of a H satisfying

$$\langle e_i, e_j \rangle = \begin{cases} 1 ; i = j \\ 0 ; i \neq j \end{cases}$$

5) The set

$$\{1, x, x^2, \dots, x^n\}$$

is the for the vector space of having degree n or less.

Exercice 06 (2 marks) : Give some properties about symmetric matrices. Answer this question in a coherent paragraph.

Good Luck

Bellaouar. Djamel

Exercise 01 (3 marks) : Read the following sentences and choose the correct item.

1. If I had found a fly in my soup I my wife.
 - Would have hit**
 - Hit**
 - I could have hit**
2. Jame's exam is tomorrow and he all day.
 - Studies**
 - Studied**
 - Has been studing**
3. Iread or write when I was four years old.
 - Can't**
 - coudn't**
 - wasn't able**
4. The sunin the west
 - Is setting**
 - Set**
 - sets**
5. It iscolder today than yesterday
 - Much**
 - Most**
 - very**
6. I've a lot of friends in the USA, butof them have visited me in Guelma.
 - Non**
 - neither**
 - both**

Exercise 02 (3 marks) : Re-write the following words in ordinary English.

/ˈræʃənəl/	/ˈpruːf/	/ˈʌnˈkɑʊntəbl/	/ˌpɒlɪˈnəʊmɪəl/
/ˈkəʊsɑɪn/	/ˈvɒljʊːm/	/ˈneglɪdʒəbl /	/ˈdemənstreɪbl/
/ˈprɒpəteɪ/	/ɪnˈkriːsɪŋ/	/mæθs /	/juːˈklɪdɪən/

Exercise 03 (1 mark) : Write the following in full form

$$\sum_{n=1}^{\infty} \|x_n\| < \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{2} \left(\frac{1}{1 - \frac{1}{2}}\right) = 1,$$

Exercise 04 (3 marks) : Type ‘the’ in gaps or ‘-’ if the is not necessary.

Selma : what's onTV tonight?

Jame : There's a new film at eight oclock, but I can't remembername of it.

Selma : I'd like to watch it. What time'sdinner?

Jame : About eight. I don't want to watch and eat at same time.

Selma : No. we can record the film and watch it later tonight.

Jame : I won't because I'm inmiddle of reading an exciting book. I want to finish it. If you record it, I'll watch it sometime next week.

Selma : Ok.

Exercise 05 (7 marks) : Translate the following paragraph in French language.

Differential equation

Mathematical statement containing one or more derivatives, that is, terms representing the rates of change of continuously varying quantities. Differential equations are very common in science and engineering, as well as in many other fields of quantitative study, because what can be directly observed and measured for systems undergoing changes are their rates of change. The solution of a differential equation is, in general, an equation expressing the functional dependence of one variable upon one or more others; it ordinarily contains constant terms that are not present in the original differential equation. Another way of saying this is that the solution of a differential equation produces a function that can be used to predict the behaviour of the original system, at least within certain constraints.

Differential equations are classified into several broad categories, and these are in turn further divided into many subcategories. The most important categories are ordinary differential equations and partial differential equations. When the function involved in the equation depends on only a single variable, its derivatives are ordinary derivatives and the differential equation is classed as an ordinary differential equation. On the other hand, if the function depends on several independent variables, so that its derivatives are partial derivatives, the differential equation is classed as a partial differential equation.

Exercise 06 (3 marks) : Give the phonetic of the following words.

History, could, wood, would, blood, responsibility, divisibility, government, procedure
Operation, equation, page, beige, partial, differential, ordinary.

Good Luck

Bellaouar. Djamel

Exercise 01 (12 marks) : Translate the following text in French language.

Metric space

In **mathematics**, especially topology, an abstract set with a distance **function**, called a metric, that specifies a nonnegative distance between any two of its points in such a way that the following properties hold: (1) the distance from the first point to the second equals zero if and only if the points are the same, (2) the distance from the first point to the second equals the distance from the second to the first, and (3) the **sum** of the distance from the first point to the second and the distance from the second point to a third exceeds or equals the distance from the first to the third. The last of these properties **is called** the triangle inequality. The French mathematician Maurice Fréchet initiated the study of metric spaces in 1905.

The usual distance function on the real number line is a metric, as is the usual distance function in Euclidean **n -dimensional space**. There are also more exotic examples of interest to mathematicians. Given any set of points, the **discrete** metric specifies that the distance from a point to itself equal 0 while the distance between any two distinct points equal 1. The so-called taxicab metric on the **Euclidean** plane declares the distance from a point (x, y) to a point (z, w) to be $|x - z| + |y - w|$. This “taxicab distance” gives the minimum **length** of a path from (x, y) to (z, w) constructed from horizontal and vertical line segments. In analysis there are several useful metrics on sets of **bounded real-valued** continuous or integrable functions.

Thus, a metric generalizes the notion of usual distance to more general settings. Moreover, a metric on a set X determines a collection of open sets, or topology, on X when a **subset** U of X is declared to be open **if and only if** for each point p of X there is a positive (possibly very small) distance r such that the set of all points of X of distance **less than** r from p is completely contained in U . In this way metric spaces provide important examples of topological spaces.

A metric space is said to be **complete** if every **sequence** of points in which the terms are eventually pairwise arbitrarily close to each other (a so-called Cauchy sequence) converges to a point in the metric space. The usual metric on the **rational** numbers is not complete since some Cauchy sequences of rational numbers do not converge to rational numbers. For example, the rational number sequence 3, 3.1, 3.14, 3.141, 3.1415, 3.14159, ... converges to π , which is not a rational number. However, the usual metric on the real numbers is complete, and, moreover, every real number is the limit of a Cauchy sequence of rational numbers. In this sense, the real numbers form the completion of the rational numbers. The **proof** of this fact, given in 1914 by the German mathematician Felix Hausdorff, can be generalized to demonstrate that every metric space has such a completion.

Exercise 02 (3 marks) : Complete the table as shown in the example.

Verb	Noun	Adjective
⇒ To integrate	⇒ integration	⇒ Integrable (integrated)
To reduce
.....	boundary
.....	derivable

Exercise 03 (2 marks) : Underline the silent letters in each of the following words.

Daughter, opera, listen, could, answer, comb, night, might, wrong, white, two, yoghurt, cheque, fruit, suit, friend.

Exercise 04 (3 marks) : Re-write the following words in ordinary English.

/ˈræʃənəl/	/pru:f/	/ˈʌnˈkɑʊntəbl/	/ˌpɒlɪˈnɒsmɪəl/
/ˈkəʊsɑɪn/	/ˈvɒljʊ:m/	/ˈneglɪdʒəbl /	/ˈdemənstreɪbl/
/ˈprɒpəti/	/ɪnˈkri:sɪŋ/	/mæθs /	/juːˈklɪdɪən/

Good Luck

Bellacuar. Djamel

Exercise 01(6marks) : Complete the following table.

French	English	Phonetic (by English)
échantillon	Sample	/ 'sɑ:mpəl /
		/ 'sætɪsfɑɪɪŋ /
Sinus x , dénominateur		
Simultanément		
Suffisant, fermeture ^f		
		/ ,ekspəʊ'nenʃəl /
Triangulaire		
Algèbre linéaire		
Inversible		
	Column	
Célèbre		
		/ ɪ'kwɪpət /, / 'meʒərəbl /
	An eigenvector	
Analyse fonctionnelle		
		/ bɔ:l /, / 'beɪsɪk /
Propriété, appliqué(e)		
Polynôme, idée		
Orthonormée		
	A basis,	
		/ 'kæpl /, / di:'kri:sɪŋ /
Analyse numérique		
Multiplicité, facilement		
Voisinage, chapitre		
Minimisation		
		/ dʒenərəlaɪ'zeɪʃən /
Valeur propre		
Hypothèse		
Ensemble, densité		
	Homogeneous, method	
		/ dɪstrɪ'bju:ʃən /

Exercise 02 (4marks) : Complete the following sentences by using the correspondent mathematical notions

1) The of M is :

$$M^\perp = \{x \in E; \langle x, m \rangle = 0 \text{ for all } m \in M\}$$

2) The

$$\|f + g\| \leq \|f\| + \|g\|$$

3) We call a bounded operator $A : H \rightarrow H$ a or operator if and only if for every x, y in H we have :

$$\langle Ax, y \rangle = \langle x, Ay \rangle$$

4) The arithmetic mean of n numbers a_1, a_2, \dots, a_n is

$$x = \frac{a_1 + a_2 + \dots + a_n}{n}$$

5) : **A property of a distance function**

$$f(x, y) \leq f(x, z) + f(z, y). \text{ For all } x, y, z.$$

6) Let (X, d) be a complete metric space and $T: X \rightarrow X$ a contraction map. Then, T has a point $x_0 \in X$; that means $T(x_0) = x_0$.

7) For a subset A of a topological space X , the smallest closed set containing A denoted \overline{A}

8) If X is a topological subspace of a metric space, compact is equivalent to a and

9) A topological space X is locally compact if every $p \in X$ has a compact

10) Let X and Y be Banach spaces and $T: X \rightarrow Y$ a bounded linear operator. T is called compact if for every bounded sequence $\{x_n\} \subset X$, $\{Tx_n\}$ has a in Y .

11) A measure μ has the property of if given A_1, A_2, \dots is a sequence of pairwise disjoint measurable sets. Then

$$\mu \left(\bigcup_{i=1}^{\infty} A_i \right) = \sum_{i=1}^{\infty} \mu(A_i)$$

Exercise o3 (4marks) : Write the following notations in full form :

$$\sum_{k=0}^{\infty} \frac{A^k}{k!}$$

$$cA = \{cx \mid x \in A\}.$$

$$\lim_{n \rightarrow \infty} \left\| \sum_1^n \alpha_i e_i \right\| = \sqrt{\sum |\alpha_i|^2}$$

$$\lim_{t \rightarrow 0} \frac{e^{At} - I}{t} = A.$$

Exercise o4 (4 marks): Translate the following sentences in English language.

a) *Dans tout le chapitre, Si X est de dimension finie, toute forme linéaire sur X est continue.*

b) *Une équation différentielle d'ordre n est linéaire ssi elle est de la forme :*

Avec quelques propriétés, on peut écrire

c) *Toute partie fermée d'un espace métrique complet est un espace métrique complet pour la métrique induite.*

d) *Considérons la suite de fonctions, montrer que les distributions $|x|$, $\text{sgn}(x)$ sont homogènes et déterminer leurs degrés respectifs. En déduire la solution dans l'espace D' , de l'équation :*

$$x T = 1.$$

Exercise o5 (2 marks) : What did you study in the **basic functional analysis**?. Give an **abstract** and answer this question in a **coherent** paragraph.

Good Luck

Bellaouar . Djamel

Exercise o1 (0,25 ×8 marks) : Re-write the following words in **ordinary** English.

[ə'prəʊtʃ], [enθ], [kɒn'vɜːslɪ],

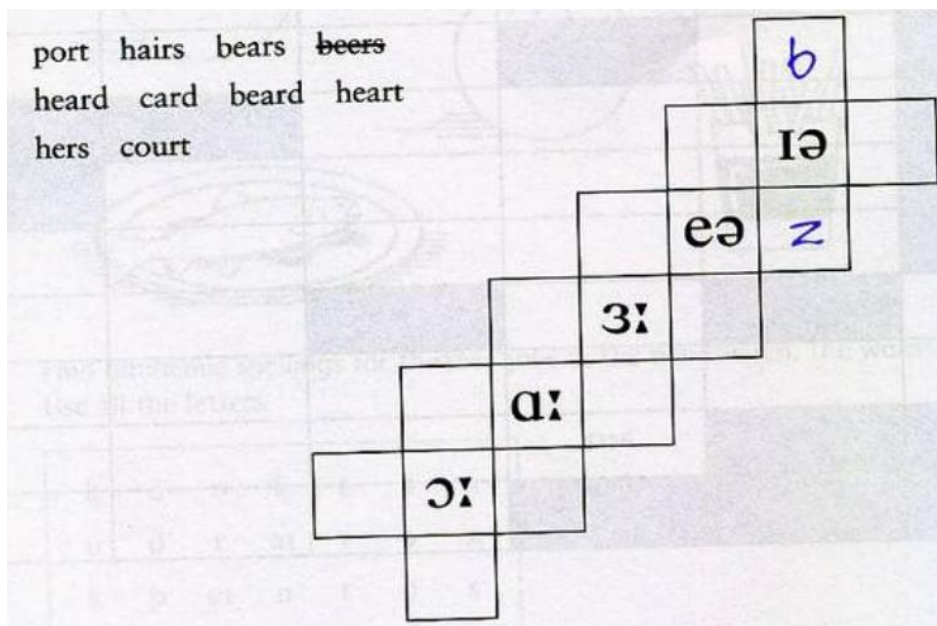
[di:'kriːsɪŋ], [ˌdɪfərənsɪ'eɪʃən],

[dɪ'vɪzəbl], [ɪn'tɪəriəʳ], ['kɜːnl],

Exercise o2 (0,25 ×8 marks) : Give the **phonetic** of the following words.

Logarithm, Measurable, Multiplicity, Neighbourhood, Infinitesimal, Quadrature, literature, future.

Exercise o3 (2 marks) : Complete the following diagram.



Exercise o4 (0,50 ×4 marks) : Finish the following sentences.

1. We will stay at home if
2. If I were a rich man.
3. I wouldn't say that to her if
4. I would hit my wife if

Exercise o5 (0,50 ×5 marks) : Fill in : **shall , will or be going to.**

A: What do you want for lunch? **B:** I think I.....have chicken and some salad.

A: John has come back from England. **B:** I know Isee him tonight.

A: I haven't got any money. **B:** Ilend you some if you want me to.

A: Have you decided where to go on holiday? **B:** yes Itravel round Europe.

A: The plants need watering. **B:** I know Iwater them later.

Exercise o6 (0,50 ×4 marks) : Put the verbs in brackets into the **correct** tense.

1. Tony (**To buy**) a new car last Monday.
2. Her eyes were red she (**To cry**).
3. What (**You/ plan**) to do after the exams?
(**You/ stay**) at home?.

Exercise o7 (2+2,5 marks) : Translate the following paragraph in *English* language.

Analyse Fonctionnelle

L'Analyse Fonctionnelle est née au début du 20-ème siècle pour **fournir** un cadre abstrait et général à un certain nombre de problèmes, dont beaucoup sont issus de la physique, et où la question posée est la recherche d'une fonction vérifiant certaines propriétés, par exemple une équation aux dérivées partielles (**EDP**).

La théorie moderne de l'intégration (**Lebesgue**, un peu après 1900) et la théorie des espaces de **Hilbert** se sont rejointes pour créer l'un des objets les plus importants, l'espace L_2 des fonctions de carré sommable, qui a permis en particulier de placer la théorie des séries de **Fourier** dans un cadre **conceptuellement** beaucoup plus clair et plus simple que celui qui était en vigueur à la fin du 19-ème siècle.

Exercise o8 (1,5 + 1,5 marks) : Translate the following paragraph in *French* language.

The Jacobi method

Abstract: *We give a general view about Jacobi's method, we describe in this work the method was discovered by Jacobi in 1846 and can used iteratively compute all the **eigenvalues** and **eigenvectors** of real symmetric matrix.*

Open problem: Let $N = 199$. We see that $N - 2n^2$ is prime whenever $2n^2 < N$. It is not known **whether** $N = 199$ is the largest **integer** with this property.

Remark: *Try by yourself. Don't be deceiver!*

Bellaouar. Dj *Good luck*

Exercise 01 (4marks): Give the **phonetic** of the following words.

Finitely, infinitely, number, unlimited, distinct, power, multiplicative, literature.

Damascus, politicization, against, Throughout, adjoint, owe, bear, near.

Exercise 02 (8marks): Translate the following paragraph in *English* language.

Intégrale Impropres

Dans l'étude de l'intégrale :

$$\int_a^b f(x) dx$$

nous avons supposé l'intervalle d'intégration compact (c'est-à-dire fermé et borné) et la fonction f bornée. Cette définition de l'intégrale est trop restreinte pour bien des applications. Dans ce chapitre, nous allons généraliser l'intégrale de Riemann en considérant des intervalles non compacts et des fonctions ne nécessairement bornées.

Soit (a, b) où $a < b$ un intervalle (ouvert, fermé ou semi-ouvert). Une fonction $f : (a, b) \rightarrow \mathbb{R}$ sera dite localement intégrable sur (a, b) si elle est Reimann-intégrable sur tout sous-intervalle compact $[a, \beta] \subset (a, b)$. On écrira dans ce cas :

$$f \in \mathbb{R}_{loc}(a, b)$$

Exercise 03 (8marks): Translate the following paragraphs in *French* language.

A Numerical solution of differential equations

Abstract. Throughout this work, our aim is to investigate both analytical and numerical techniques for studying the solution of differential equations.

Abstract. We prove the uniqueness of the solution of some operator differential equations with constant periodic coefficients with the help of Green's function, the operator coefficients are unbounded and their domain and range belongs the Hilbert space.

Self adjoint compact operators

Abstract. We call a bounded operator $A : H \rightarrow H$ a self-adjoint or symmetric operator if and only if for every x, y in H , we have :

$\langle Ax, y \rangle = \langle x, Ay \rangle$. We start with a few general properties of such operator. We present here the main properties of the self-adjoint operators.

Remark: Try by yourself. Don't be deceiver!

Bellacour Dj

Good luck

Exercise 01 (10marks): **A)** Turn from active to passive the following statements:

1. In this chapter **we will prove** the contraction mapping theorem.
2. We also **present** some applications.
3. Hilbert **had let** many problems without proof.
4. We **may use** the contraction mapping theorem to prove the existence and uniqueness of solutions.

B) Put the verbs in brackets into the correct tense:

1. If she(not break) the window, she wouldn't have had to pay for a new one.
2. If it(not be) cold, they wouldn't have lit the fire.
3. If she studied more, she(be) a better student.
4. If I lived in America, I(speak) English well.

C) Give the **phonetic** of the following words.

Assumption, Absolutely, Binomial, characterization,
Comparison, Conjecture, Diagonalizable, Homogeneous

Exercise 02 (10marks): Translate the following paragraphs in *French* language.

Complete closed subspaces in a seminormed space

Abstract. The absolute value function on \mathbb{R} and the modulus on \mathbb{C} are denoted by $|\cdot|$ and each gives a notion of length or distance in the corresponding space and permits the discussion of convergence of sequences in that space or continuity of functions on that space. In this work, we shall extend these concepts to a general linear space E .

Abstract. A seminorm on the linear space E is a function $p: E \rightarrow \mathbb{R}$ for which $p(\alpha x) = |\alpha| p(x)$ and $p(x+y) \leq p(x) + p(y)$ for all $\alpha \in \mathbb{K}$ and $x, y \in E$. The pair (E, p) is called a seminormed space. We study some properties concerning seminormed spaces, for example, a closed subspace of a seminormed space is complete but the reciprocal is false. Finally, we prove that a complete subspace of a normed space is closed.

Key words and phrases. Functional analysis, seminormed spaces, normed spaces, subspaces, completeness.

Remark: Try by yourself. Don't be *deceiver!*

Bellaouar. Dj

Good luck

Exercise 01 (5 marks): Translate the following sentences in English language.

- a) Autrement dit, il suffit de prouver que. Il vient donc
- b) Considérons un opérateur différentiel linéaire à coefficients constants
- c) La condition est nécessaire, puisque pour toute fonction $\phi \in D$, on a
- d) **Théorème** : Toute suite convergente dans un espace métrique est une suite de Cauchy.
- e) Une application vérifiant les propriétés suivantes :
- f) Un voisinage d'un point a est une partie de E contenant une boule ouverte centrée en a .
- g) **Théorème de Riesz**. Si la boule unité d'un **evn** $(E, \|\cdot\|)$ est compact, alors, E est de dimension finie.
- h) **Exercice** : Une application multilinéaire continue entre un produit d'espaces vectoriels normés et un espace vectoriel normé est lipschitzienne sur chaque sous-ensemble borné.
- i) Nous obtenons donc la formule générale, d'après l'hypothèse, on trouve
- j) Donc le *sup* existe. Inversement, soit E un espace de Hilbert muni d'une base hilbertienne.

Exercise 02 (3 marks): Read the following sentences and choose the correct item.

- How would you feel if youyour car?
 - **crash**
 - **will crash**
 - **crashed**
- If I were you, Ito bed early.
 - **will go**
 - **would go**
 - **won't go**
- Wasman who robbed the bank arrested?
 - **a**
 - **an**
 - **the**
- I'm hungry but there'sin the fridge for me to eat.
 - **anything**
 - **nothing**
 - **something**
- Their house isthan ours.
 - **big**
 - **bigger**
 - **biggest**

- Don't worry. I'm sure heto you soon.
 - writes
 - will write
 - would write

Exercise 03 (3 marks): Re-write the following words in ordinary English.

/ˈɒbvɪəs/	/'nɜ:vəs/	/dɪs'kʌs/	/,pɒlɪ'nəʊmɪəl/
/'prɪ:vɪəs/	/'sɪərɪəs/	/ɪn'kri:sɪŋ/	/'neɪbəhʊd/
/'peɪfənt /	/'speɪfəs /	/'ʌn'kaʊntəbl/	/ju:'klɪdɪən/

Exercise 04 (2.5 marks) : Give the phonetic of the following words and phrases.

- ✓ Height, weight, suggest a substitution, some questions
- ✓ Operator, temperature, future and literature.

Exercise 05 (1,5 mark) : Write the following in full form.

$$M_n(\mathbb{R}) = S_n(\mathbb{R}) \oplus A_n(\mathbb{R})$$

(i)

$$\lim_{t \rightarrow 0} \frac{e^{At} - I}{t} = A.$$

(ii) Prove that the

$$\left\| \frac{A^k}{k!} \right\| \leq \frac{\|A\|^k}{k!}$$

(iii) Show that the

Exercise 06 (3 marks): Complete the following sentences by using the correspondent mathematical notions.

1. f is said to be a on $[a, b]$ if there exists a constant L such that $0 < L < 1$ and

$$|f(x) - f(y)| \leq L|x - y|; \forall x, y \in [a, b]$$

2. We may denote a by the notation

$$f : X \rightarrow Y, \quad x \mapsto f(x),$$

3. The special notation \emptyset is reserved for the set, the set with no elements. The set is a subset of any set.

4. and are the most basic concepts of mathematics. Given any x and any X , either x belongs to X (denoted $x \in X$), or x does not belong to X (denoted $x \notin X$).

5. Let (X, d) be a metric space. An of radius $\varepsilon > 0$ centered at a is

$$B_d(a, \varepsilon) = \{x : d(x, a) < \varepsilon\}$$

6. If X is and $f : X \rightarrow Y$ is continuous, then, $f(X)$ is compact.

7. A subset of \mathbb{R}^n is compact \Leftrightarrow the subset is and

Exercise 07 (2 marks) : Choose only one of the following topics.

I : Translate the following paragraph in Arabic or French language.

Topological Basis

The key topological concepts and theories for metric spaces can be introduced from balls. In fact, if we carefully examine the definitions and theorems about open subsets, closed subsets, continuity, etc., then we see that we have used exactly two key properties about the balls. Whether or not we have a metric, as long as we have a system of balls satisfying these two properties, we should be able to develop similar topological theory. This observation leads to the concept of topological basis.

II : What did you study in the **basic numerical analysis**? Give an **abstract** and answer this question in a **coherent** paragraph.

Good Luck

Bellacuar Djamel

Exercise o1 (0.25×24 marks): Translate the following sentences in English language.

1. Déterminer si les ensembles suivants sont bornés.
2. Cas le plus général d'espace topologique.
3. Fonctions complexes et continuité.
4. Continuité et limite dans les espaces métriques ou normés.
5. Le théorème principal sur les résidus.
6. Polynômes et fonctions rationnelles.
7. Soit N un entier positive suffisamment grand.
8. Topologie et approximation de fonctions caractéristiques.
9. Dans la prochaine section on présentera une des applications les plus importantes du Théorème 1.
10. Comment déterminer un rayon de convergence ?
11. **Définition** . Le complémentaire d'un sous ensemble ouvert de X sera appelé sous ensemble fermé.
12. **Proposition**
 - X et \emptyset sont fermés.
 - Une réunion finie de fermés est fermée.
 - Une intersection quelconque de fermés est fermée.
13. D'autre part, il existe un élément $x \in E$ tel que.
14. Opérations élémentaires sur les distributions.
15. Table des matières.
16. Polynômes orthogonaux, Orthogonalité.
17. Tout polynôme positif est somme de deux carrés.
18. Considérons ensuite la fonction f définie par :
19. Plus généralement, on a le résultat suivant.
20. Si Ω est un ouvert borné à frontière **lipschitzienne**, alors
21. Groupe orthogonal réel et groupe spécial orthogonal réel .
22. Quelques résultats supplémentaires d'arithmétique et théorie des nombres.
23. En appliquant systématiquement cette formule, nous obtenons

24. **Remarque**. D'une façon plus générale, on peut prouver que les applications f, g sont linéaires continues pour la convergence dans D .

Exercise o2 (0.25×8 marks): Underline the correct item

1. John and Selma are listening to music **every day** / **at the moment**.
2. He **bought** / **has bought** a new computer last week.
3. I've lived here **since** / **for** 1990.
4. She usually **is visiting** / **visits** her grandparents on sundasy.

5. This exercise is very **easily** / **easy**
6. Bob is the best student **of** / **in** our class.
7. The **chair's leg** / **leg of the chair** is broken
8. That's the house **where** / **which** I grew up

Exercise o3 (0.125×7 mark): Fill in : **who, whose, what, where, when, why, which**

- are you looking for? My keys
 do you live? In Guelma
 is your car? The blue one
 was she angry? Because someone had stolen her bag
 is Mister John? The new English teacher
 will you come back? Next Friday
 is this suitcase? I don't know.

Exercise o4 (0.25×8 marks): Add question tags to the following statements

- He likes apples, doesn't he?
- She doesn't like apples, does she?
- He never understands, does he?
- She is sleeping, isn't she?
- He came too late, didn't he?
- He didn't come too late, did he?
- Let him come with us, won't you?
- There is no one here, is there?

Exercise o5 (0.125×15 marks): Re-write the following words in **ordinary** English.

- ['sentəns], [kən'tɪnjʊəs], [sɪməl'teɪnɪəs], ['speɪfəs]
 ['deɪndʒrəs], [glʌv], [dɪ'sɪʒən]
- fræŋk, faʊnd, fɔːr, frɒg, 'lɑːfɪŋ, ɒn, ðə, flɔːr

Exercise o6 (0.25×22 marks): Complete the following sentences by using the correspondent mathematical notions

1. \mathbb{N} The set of
2. \mathbb{Q} The set of
3. \mathbb{R} The set of
4. Consider a square matrix A . A nonzero vector x is an of the matrix with ℓ if $Ax = \ell x$.
5. A has the general form

$$p_n(x) = a_0 + a_1x + \dots + a_nx^n$$

6. Every of a convergent sequence converges to the same limit.

7. The with (1,1) and 3 has the equation

$$(x - 1)^2 + (y - 1)^2 = 9$$

8. **Theorem :** A nonempty set S of reals which is bounded above has the

9. Show that the **series**

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

is

10. Let \mathcal{R} be an equivalence relation on a set A . That means

\mathcal{R} (1) is, (2) is, (3) is

11. The interior of A is the union of all open sets in A

12. The closure of A is the intersection of all closed sets A

13. A vector v is a of the vectors x, y and z if it can be written as

$$v = \alpha x + \beta y + \gamma z ;$$

where α, β, γ are constants.

14.....**Set**

15. A set of a space H satisfying :

$$\langle e_i, e_j \rangle = \begin{cases} 1 ; i = j \\ 0 ; i \neq j \end{cases}$$

16. A subset E of a vector space V is called a of V if each vector $x \in V$ can be uniquely written in the form

$$x = \sum_{i=1}^n \alpha_i e_i ; e_i \in E$$

Exercise 07 (1.57 marks) : What did you study in the **basic Algebra (1, 2, 3 and 4)**? Give an **abstract** and answer this question in a **coherent** paragraph.

Bellacuar Djamel

Good Luck

Exercise 01 (0.25×16 marks):

A) Complete the following sentences.

1. Last night I (to lose) my keys. I had to call my brother to let me in.
2. I (to lose) my keys. Can you help me look for them?
3. I (to visit) Paris three times.
4. I (drink) three cups of coffee this morning.

B) Complete the following conjugation by using the verb (to see).

He sees	He is seeing	He	He has been seeing
He	He	He	He
He	He	He	He
He	He	He	He

Exercise 02 (0.50×8 marks): Change the direct speech into reported speech. Choose the past simple of ‘ask’, ‘say’ or ‘tell’:

1. “Come quickly!”

She

2. “Did you arrive before seven?”

She

3. “I usually drink coffee in the mornings”

She.....

4. “I’ll come and help you on Saturday”

She.....

5. “I would have visited the hospital, if I had known you were sick”

She

6. "I'll come and help you at twelve"

She

7. "What are you doing tomorrow?"

She

8. "I've never been to Wales"

She.....

Exercise 03 (0.25×12 marks): Re-write the following words in **ordinary** English.

[pə'zɪfən]	['kʌrɪdʒ]	[ɪn'ʃʊərəns]
[pə'zeɪʃən]	[kəm'pæfən]	[æm'bɪfəs]
[ə'ʃʊərəns]	['fɔ:tʃənɪtlɪ]	['preʃəs]
['præktɪs]	['præktɪs]	['kɒnfəs]

Exercise 04 (0.25×12 marks): Complete the following sentences by using the correspondent mathematical notions.

- f is said to be a on $[a, b]$ if there exists a constant L such that $0 < L < 1$ and

$$|f(x) - f(y)| \leq L|x - y|; \forall x, y \in [a, b]$$

- We may denote aby the notation

$$f : X \rightarrow Y, \quad x \mapsto f(x),$$

- The special notation \emptyset is reserved for the set, the set with no elements. The set is a subset of any set.
- and are the most basic concepts of mathematics. Given any x and any X , either x belongs to X (denoted $x \in X$), or x does not belong to X (denoted $x \notin X$).

- Let (X, d) be a metric space. An of radius $\varepsilon > 0$ centered at a is

$$B_d(a, \varepsilon) = \{x : d(x, a) < \varepsilon\}$$

- If X is and $f: X \rightarrow Y$ is continuous, then, $f(X)$ is compact.
- A subset of \mathbb{R}^n is compact \Leftrightarrow the subset is and

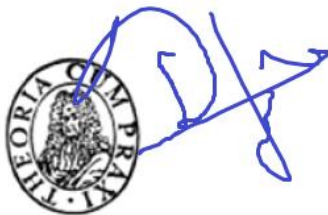
Exercise 05 (0.5×12 marks): Translate the following sentences in English language.

1. Déterminer si les ensembles suivants sont bornés.
2. Continuité et limite dans les espaces métriques ou normés.
3. Le théorème principal sur les résidus.
4. Polynômes et fonctions rationnelles.
5. Soit N un entier positive suffisamment grand.
6. Topologie et approximation de fonctions caractéristiques.
7. Dans la prochaine section on présentera une des applications les plus importantes du Théorème 1.
8. Comment déterminer un rayon de convergence ?
9. *Définition*. Le complémentaire d'un sous ensemble ouvert de X sera appelé sous ensemble fermé.
10. *Proposition*.
 - X et \emptyset sont fermés.
 - Une réunion finie de fermés est fermée.
 - Une intersection quelconque de fermés est fermée.
11. Si Ω est un ouvert borné à frontière *lipschitzienne*, alors
12. Quelques résultats supplémentaires d'arithmétique et théorie des nombres.

Exercise 06 (1,5 marks) : Write the following mathematical notations in full form.

$$\lim_{t \rightarrow 0} \frac{e^{At} - I}{t} = A.$$

$$\sum_{k=1}^{\infty} \frac{1}{(2k+1)^2} = \frac{\pi^2}{24} - 1 \quad \frac{\partial^2 f}{\partial x^2}$$



Bellaouar Djamel

GOOD LUCK

Exercise 01 (0.5×8 marks): Complete the following table

word	opposite	word	opposite
Countable		Empty	
Decreasing		Bounded	
Logarithm		Commutative	
Prime Number		Homogeneous	

Exercise 02 (12 marks): Read carefully the following text.

Number theory (or arithmetic) is a branch of pure mathematics devoted primarily to the study of the integers, sometimes called "The Queen of Mathematics" because of its foundational place in the discipline. Number theorists study prime numbers as well as the properties of objects made out of integers (e.g., rational numbers) or defined as generalizations of the integers (e.g., algebraic integers). Many questions regarding prime numbers remain open, such as Goldbach's conjecture (that every even integer greater than 2 can be expressed as the sum of two primes), and the twin prime conjecture (that there are infinitely many pairs of primes whose difference is 2). Such questions spurred the development of various branches of number theory, focusing on analytic or algebraic aspects of numbers. Primes are used in several routines in information technology, such as public-key cryptography, which makes use of properties such as the difficulty of factoring large numbers into their prime factors. Prime numbers give rise to various generalizations in other mathematical domains, mainly algebra, such as prime elements and prime ideals.

Questions:

1. Give a suitable title of the text. Complete the following sentence:

Mathematics is theof sciences and Number Theory is theof Mathematics.

2. Find a word or expression in the text which, in context, is similar in meaning to :

Theoretician, different, to stay, piece of news, essentially.

3. Give the phonetic of the text.

Exercise 03 (1×4 marks): Change the direct speech into reported speech.

1. "I won't vote at the next election." She said
2. "Richard won't drink coffee." She said
3. "We ate Chinese food, then we walked home." She told me
4. "She didn't buy the dress." He told me

Good Luck

Bellaouar Djamel

Exercise 1 (2 marks). Write the phonetic of :

- Which child put chalk on the teacher's chair ?
- Frank found four frogs laughing on the floor.

Exercise o2 (2 marks)

Add **ed** to the verbs and put them in the correct column: ‘cry, stay, stop, hate, taste, prefer, fry, dance, certify, apply, equip, travel, dry, annoy, enjoy, occupy, realize, oppose, serve, stop, play, refuse, destroy, cut, come, end.

+ d	+ ied	+ ed	Double consonant + ed

Exercise 3 (2 marks). What is the time ?

9:00 : It is nine o'clock.

9:05, 8:55, 9:10, 8:50, 8:45, 9:20, 8:40, 9:25, 8:35, 9:30, 13:58.

Exercise 4 (9 marks). Write the following formulas in full form :

$$x_n \xrightarrow{n \rightarrow \infty} 0$$

$$\lim_{x \rightarrow 0} \frac{f''(x)}{F''(x)} = \lim_{x \rightarrow 0} \frac{-e^x}{4} = -\frac{1}{4}.$$

$$r = \sqrt{x^2 + y^2}$$

$$A \sim B \implies e^A \sim e^B$$

$$A_n = \{x \in A \mid x \leq n\}$$

$$\left\| \frac{A^k}{k!} \right\| \leq \frac{\|A\|^k}{k!} \quad A \neq \emptyset \quad p \notin R.$$

$$\left| \sum_{k=1}^n x_k \right| \leq \sum_{k=1}^n |x_k|. \quad A_1 \times A_2 \times \cdots \times A_n$$

$$\langle f, g \rangle = \int_a^b f(x) g(x) dx$$

- ◆) $\forall x \in E : \|x\| \geq 0$, et $\|x\| = 0 \Leftrightarrow x = 0$
- ◆) $\forall \lambda \in \mathbb{K}, \forall x \in E : \|\lambda x\| = |\lambda| \cdot \|x\|$
- ◆) $\forall x, y \in E : \|x + y\| \leq \|x\| + \|y\|$.

$$\mathbb{M}_n(\mathbb{R}) = S_n(\mathbb{R}) \oplus A_n(\mathbb{R})$$

$$|gf| = gf \text{ and } \left(\frac{|g|}{\|g\|_q} \right)^q = \left(\frac{|f|}{\|f\|_p} \right)^p \text{ a.e.}$$

Exercise 05 (5 marks): Translate the following sentences in English language.

1. Notre premier objectif est de munir \mathbb{R} d'une structure de corps commutatif. Rappelons que \mathbb{Q} désigne le corps des nombres rationnels.
2. Où c est une contante arbitraire. De (1) et (2) on déduit que
3. La solution (3) s'appelle solution de D'Alembert.
4. Équation différentielle linéaire homogène d'ordre supérieur.

Part 10. Phrases and sentences used in mathematical papers

By Dr. Bellaouar Djamel, University of 08 Mai 1945 Guema.

email: bellaouar.djamel@univ-guelma.dz

1. Since w_d is periodic mod $[n_1, n_2, \dots, n_k]$, it suffices to show $w_d(m) = 0$ for any $m \in \mathbb{N}$. Therefore, ...
2. Let us now characterize the covering equivalence of two systems of arithmetic sequences.
3. By comparing the coefficients of powers of z , we find that $w_d = w_a$ if and only if ...
4. Letting $x \rightarrow +\infty$ we obtain from (12) and (13) that
5. Hence, by Lemma 1, we get

$$\prod_{i=1}^n f(x_i, y) = 0,$$

which contradicts (3).

6. By the assumption of the lemma, we have
7. Multiplying the both sides of the equality obtained above by A^{-1} , we get
8. As a consequence, one of the following statements

(a) $\delta^{1,1,i}(n) = 2$

(b) $\delta^{1,1,i}(n) = 3$

holds

9. Theorem 16.1 and Theorem 16.2 show that a set A with $\gcd(A) = 1$ has positive density α if and only if $\log p_A(n) \simeq c\sqrt{n}$. Erdős proved these results in his paper [12], where Theorem 16.3 is also stated and applied.
10. This completes the proof. \square This completes the proof of Theorem 1.2. \square
11. Suppose that $u_i = 0$ for some i . Then
12. There exists a unique nonnegative integer m such that
13. Let

$$f(x) = \sum_{i=1}^n a_i x^i$$

be a polynomial of degree n with complex coefficients. Then

14. Choose $P \geq 1$ such that
15. **Proof.** The proof is by induction on the degree n of the polynomials.

16. Let $k \geq 2$, and assume that the theorem holds for $s' = s(k - 1)$ polynomials of degree $k - 1$. Define
17. Since $[x] \leq x < [x] + 1$ for every real number x , we have
18. Applying Lemma 12.1 to $S = B_1 + B_2$, we obtain
19. For $j = 1, \dots, s$, let
20. By Lemma 2.2, By Lemma 1.4, we have
21. We can express the difference set D'_q as follows
22. We shall use the induction hypothesis for polynomials of degree $k - 1$ to obtain an upper bound for the weight of v .
23. Since $s < 2t - 1$, we obtain the following upper bound for the weight of v
24. This section introduces analysis on finite abelian groups and their characters. We begin by using elementary number theory to determine the structure of finite abelian groups.
25. For all functions $f \in L^2\left(\frac{\mathbb{Z}}{2\mathbb{Z}}\right)$,
26. The vector space $L^2(G)$ has a basis $\{\delta_k\}_{1 \leq k \leq n}$, where the delta function δ_k is defined by
27. We shall compute the matrix of the linear operator F with respect to this basis. We have
28. Therefore, the matrix of F with respect to the basis $\{\delta_k\}_{1 \leq k \leq n}$ is
29. We can compute the exponent of ω as follows
30. Since

$$\Psi(x) \leq \int_{\Delta} f(t) dt,$$
 it follows that
31. Thus, for every $\varepsilon > 0$, the function $\Psi_\varepsilon(a, b, c)$ is bounded above, and this is equivalent to the *abc* conjecture. This completes the proof.
32. For every positive integer n , define
33. For every positive integer n and prime p ,
34. **Proof.** We know that m divides a^k if and only if $v_p(m) \leq v_p(a^k) = kv_p(a)$ for every prime p . If there exists an integer k such that m divides a^k , then $v_p(a) > 0$ whenever $v_p(m) > 0$, and so every prime that divides m also divides a .

35. **Theorem (Euclid's Theorem).** *There are infinitely many primes.*

Preuve. Let p_1, p_2, \dots, p_n be any finite set of prime numbers. Consider the integer

$$N = p_1 p_2 \dots p_n + 1.$$

Since $N > 1$, it follows from the fundamental theorem of arithmetic that N is divisible by some prime p . If $p = p_i$ for some $i = 1, \dots, n$, then p divides $N - p_1 p_2 \dots p_n = 1$, which is absurd. Therefore, $p \neq p_i$ for all $i = 1, \dots, n$. This means that, for any finite set of primes, there always exists a prime that does not belong to the set, and so the number of primes is infinite. ■

1 How to use ; ?

1. **Definition.** Let X be a standard set, and let $(A_x)_{x \in X}$ be an internal family of sets.

- A union of the form $G = \bigcup_{x \in X} A_x$ is called a *pregalaxy*; if it is external G is called a *galaxy*.
- An intersection of the form $H = \bigcap_{x \in X} A_x$ is called a *prehalo*; if it is external H is called a *halo*.

2. If a or b is in \mathbb{N} , then $F_{a,b}(z)$ becomes a polynomial; otherwise $F_{a,b}(z)$ has radius of convergence 1.

2 Hence

1. Every prime that divides a is less than $\log_4 x$, and, by condition (i), every prime power that divides n , and **hence** a , is also less than $\log_4 x$. Since $\omega(a) \leq \omega(n) < 5y$ by condition (ii), it follows that

$$1 \leq a < (\log x)^{20y}.$$

Therefore,

$$d = d'q' = dq'q',$$

and so $qq' = 1$, **hence** $q = q' = \pm 1$ and $d = \pm d'$. Since d and d' are positive, we have $d = d'$, and d is the unique positive integer that generates the subgroup H .

2. It follows that every common divisor of A must divide d , **hence** d is a greatest common divisor of A .

3. Obviously the last formula holds for $h = 0$ as well. **Hence** the latter assertion of the lemma follows from (3.4).

4. **Hence** the estimate (1.5) holds whenever $|f(x) - \alpha| \leq \varepsilon$, $\alpha \in A$ and $x \in B$.

5. **Hence** it follows from (4.32) that

6. **Hence** our situation belongs to the linear sieve problems.

7. **Hence** it follows from (6.27), (6.26) and (6.1) that

8. **Hence**, by using mainly Lemmata 2.1, 2.2, 3.3, 3.7, 4.6 and 6.1, we can establish the upper bound

9. **Hence**, we get **Hence**, **Hence**, we have Hence we have

10. **Hence**, by Lemma 2,

11. **Hence**, by Lemma 4, we get **Hence** Theorem 1 gives

12. **Hence**, exchanging j and k , we have

13. **Hence**, applying the results of Deligne and Weinstein, we can estimate the error term in the asymptotic formula for $\sum_{n=0}^{+\infty} c_n$.

14. **Hence** we may take in Theorem 1,

$$\varepsilon = \frac{1}{10} \text{ and } H = 2.$$

15. **Hence**, using Stirling's formula and Lemma 2 of [2] we can show that

16. **Hence** the proof of Theorem 2 is complete.

3 Involving

1. For the covering equivalence between systems in $S(\mathbb{R})$, Erdős [5] provided some characterizations **involving** Euler polynomials and recursions for Euler numbers.
2. You can use the word "involving" in your article as a title, For example, : <<Identities **involving** covering systems>> or <<On congruences involving Bernoulli numbers and the quotients of Fermat and Wilson>> or <<Identities **involving** the coefficients of a class of Dirichlet series>>.
3. The second example gives a generalization of a series **involving** the Hurwitz-zeta-function, which may have applications in zeta-regularization theory.
4. Our treatment of integrals **involving** $G_2(a)$ or its kind is motivated by the proof of Lemma 2 of Bridern [2].

4 Otherwise

1. Otherwise, this case is reduced to the case $s \leq n - 2$.
2. Therefore,

$$\frac{d(q^\alpha)}{q^{\alpha\gamma}} \leq \begin{cases} 0, & \text{if } \alpha \text{ is rational,} \\ 1, & \text{otherwise.} \end{cases}$$

3. If a or b is in \mathbb{N} , then $F(z)$ becomes a polynomial, otherwise $F(z)$ has radius of convergence 1.

4. If a or b is in \mathbb{N} , then $F(z)$ becomes a polynomial; otherwise $F(z)$ has radius of convergence 1.

4. If some d divides n , then n is composite; otherwise, n is prime.

5. **Definition.** Let $k = 2$ and $(a, m) = 1$. If the congruence $x^2 \equiv a \pmod{m}$ is solvable, then a is called a quadratic residue modulo m . Otherwise, a is called a quadratic nonresidue modulo m .

6. Define the von Mangoldt function

$$\Lambda(n) = \begin{cases} \log p & \text{if } n = p^k \text{ is a prime power,} \\ 0 & \text{otherwise.} \end{cases}$$

1. We must have $Lf = 0$, for **otherwise** we can replace f by $f - Lf$.
2. We claim that $f(z) > 1$. **Otherwise**, the disc D would intersect B .
3. We now prove..... Indeed, suppose **otherwise**. Then.....
4. Unless **otherwise** stated, we assume that.....
5. Moreover, for L tame or **otherwise**, it may happen that E is a free module.
6. Simplicity (or **otherwise**) of the underlying graphs will be discussed in the next section.

5 it follows that

1. Since

$$\zeta(2) = \sum_{n=1}^{+\infty} \frac{1}{n^2} = \frac{\pi^2}{6},$$

it follows that

$$\frac{1}{\zeta(2)} = \sum_{n=1}^{+\infty} \frac{\mu(n)}{n^2} = \frac{6}{\pi^2}.$$

2. Since

$$\{\max(k_i, l_i), \min(k_i, l_i)\} = \{k_i, l_i\}, \text{ for } i = 1, 2, \dots, n$$

and since f is multiplicative, it follows that

...

3. Since $\mu(d) = 0$ if d is not square-free, it follows that

4. **Proof.** Since $\lim_{k \rightarrow +\infty} f(p^k)$, it follows that there exist only finitely many prime powers p^k such that $|f(p^k)| \geq 1$, and so we can define

$$A = \prod_{|f(p^k)| \geq 1} |f(p^k)|.$$

Then $A \geq 1$.

5. Since $d(p^a) = a + 1$, it follows that

6. **Theorem (Euler).** An even integer n is perfect if and only if there exist prime numbers p and q such that $q = 2^p - 1$ and $n = 2^{p-1}q$.

Proof. If n is of this form, then q is odd and $2n = 2^p q$. It follows that

$$\begin{aligned}\sigma(n) &= \sigma(2^{p-1}) \sigma(q) \\ &= (2^p - 1)(q + 1) \\ &= 2^p q + (2^p - q - 1) \\ &= 2n,\end{aligned}$$

and so n is perfect.

6 of the form

1. It follows that every solution of the equation $ab + cd = n$ is **of the form** $b = b_0 + \gamma h$ and $d = d_0 - \alpha h$ for some integer h .

2. If the integers b_0 and d_0 solve the equation $ab + cd = n$, then every solution is **of the form**

$$b = b_0 + \gamma h$$

and

$$d = d_0 - \alpha h$$

3. There are many beautiful open problems about prime numbers. Here are some examples. Do there exist infinitely many primes **of the form** $n^2 + 1$. For example, $5 = 2^2 + 1$, $17 = 4^2 + 1$, and $101 = 10^2 + 1$. The best result is due to Erdős [3], who proved that there exist infinitely many integers n such that $n^2 + 1$ is either prime or the product of two primes.

4. Every such integer is of the form $n = 2.3.5\dots p_k$.

5. A Dirichlet series is a function **of the form**

$$F(s) = \sum_{n=1}^{\infty} \frac{a_n}{n^s},$$

where $\{a_n\}_{n=1}^{\infty}$ is a sequence of complex numbers.

6. Let A and B be nonempty sets of integers. The sumset $A + B$ is the set consisting of all integers of the form $a + b$, where $a \in A$ and $b \in B$. The difference set $A - B$ consists of all integers of the form $a - b$, where $a \in A$ and $b \in B$.

7. Prove that every multiple of 6 can be written as the sum of a bounded number of integers of the form $x(x-1)(x-2)$ with $x \in \mathbb{N}_0$.

8. Rademacher [1] obtained a convergent series for $p(n)$ of the form

$$p(n) = \dots$$

9. Consider the set S of nonnegative integers of the form

$$a - dx,$$

where $x \in \mathbb{Z}$.

10. For example, if H is the subgroup consisting of all integers of the form $35x + 91y$, then $7 = 35(-5) + 91(2) \in H$ and $H = 7\mathbb{Z}$.

11. Let H be the subset of \mathbb{Z} consisting of all integers of the form

7 in the form

1. Conversely, if n is an even perfect number, then $\sigma(n) = 2n$. Writing n **in the form**

2. Let a and b be integers with $b \geq 1$. There is a simple and efficient method to compute the greatest common divisor of a and b and to express (a, b) explicitly in the form $ax + by$.

3. If the positive integer n is composite, then n can be written in the form $n = dd'$, where $1 < d \leq d' < n$.

4. Show that every integer $n \geq 33$ can be written in the form $n_0 + 6k$ for some nonnegative integer k and $n_0 \in \{33, 34, \dots, 38\}$.

5. Prove that the number of positive integers $n \leq x$ that can be written in the form

6. Let $N(a_1, a_2)$ denote the number of nonnegative integers that cannot be represented in the form

$$a_1x + a_2y$$

with x, y nonnegative integers.

7. If $f(t)$ has degree n with leading coefficient a_n , then $f(t)$ factors uniquely in the form

$$f(t) = \dots$$

8. Prove that we can factor m uniquely in the form $m = m_0m_1$, where $(m_0, m_1) = 1$.

9. Since Mason's theorem is symmetric in a, b and c , we could also write the equation in the form $a + b + c = 0$.

10. Every real number x can be written uniquely in the form $x = [x] + \{x\}$.

11. Prove that every positive integer n can be written uniquely in the form $n = k^2\ell$, where k and ℓ are positive integers and ℓ is square-free.

8 denote, denotes and denoted by

1. Let $\log_2 x$ **denote** the logarithm of x to the base 2.

2. We **denote by** $\frac{\mathbb{Z}}{m\mathbb{Z}}$ the set of all congruence classes modulo m .

3. We **denote** the inverse of a by a^{-1} .

4. Let $N_m(x)$ **denote** the number of positive integers not exceeding x that are relatively prime to m .

5. The order of a modulo m , **denoted by** or $d_m(a)$, is the smallest positive integer d such that ...

6. The degree of the polynomial $f(x)$, **denoted by** $\deg(f)$, is the greatest integer n such that $a_n \neq 0$, and a_n is called the leading coefficient.

7. This integer k is called the index of a with respect to the primitive root g , and is **denoted by**

$$k = \text{ind}_g(a).$$

8. , where b **denotes** the cyclic subgroup of \mathbb{Z} .

9. , where $N_0(abc)$ **denotes** the number of distinct zeros of the polynomial abc , and $\text{rad}(abc)$ is the radical of abc .

10. , where $N(x)$ **denotes** the set of all positive integers n divisible only by primes $p \leq x$.

11. Recall that $[x]$ **denotes** the integer part of the real number x .

12. If A_1, A_2, \dots, A_n are n sets of integers, then

$$A_1 + A_2 + \dots + A_n$$

denotes the sumset consisting of all integers of the form $a_1 + a_2 + \dots + a_n$, where $a_i \in A_i$ for $i = 1, 2, \dots, n$.

13. If A is a nonempty set of integers, then $\gcd(A)$ **denotes** the greatest common divisor of the elements of A .

14. Let $G = \{2\mathbb{Z}, 1 + 2\mathbb{Z}\}$, where $2\mathbb{Z}$ denotes the set of even integers and $1 + 2\mathbb{Z}$ the set of odd integers.

15. Define the map $f : \mathbb{R} \rightarrow \mathbb{R}/\mathbb{Z}$ by $f(t) = \{t\}$, where $\{t\}$ **denotes** the fractional part of t .

16. In (2.1), f_n **denotes** the n -th Fibonacci number.

17. As usual, $\theta = [a_0, a_1, a_2, \dots]$ **denotes** the simple continued fraction of θ .

18. In what follows, $\sigma(A)$ **denotes** the set of eigenvalues of a square matrix A .

19. , which **denotes** the set of all finite words over \mathbb{K} in the usual sense.

20. The right-hand side given above **denotes** a 3-dimensional word of size 4.

21. In this section, $s \geq 1$ **denotes** a fixed integer.

22. , where $B(r, a)$ **denotes** the open ball $\{x \in \mathbb{R}^s : \|a - x\| < r\}$.

23. , where ρ **denotes** the complex conjugation in \mathbb{C} .

9 Using

1. **Using** (1), (5), (7) and $Q_k > 1$, we get

2. **Using** these powerful necessary conditions, we get

3. **By using** (3.5), (3.6) and (3.7), we have

4. It follows that

$$A = \prod_{|f(p^k)| \geq 1} |f(p^k)|,$$

by using (2.3).

5. Thus, **using** (4.8) again, we obtain

6. Hence, **by using** mainly Lemmata 2.1, 2.2, 3.3, 3.7, 4.6 and 6.1, we can establish the upper bound

7. Now, **using** the Riemann-von Mangoldt formula, we see that for any $a > 0$ and for all $T > T_0$,

8. **Using** integration by parts, we have

9. Now, **using** the well-known formula

10. Using the same method employed in Section 3, we can calculate the leading coefficient of the polynomial $P_n(z)$, namely,

10 Throughout

1. **Throughout** this section we denote Euler's constant by γ .

2. **Throughout** this paper, ε is an arbitrarily small positive constant.

3. **Throughout** this paper, the empty sum is to be considered as zero.
4. **Throughout** this paper except in the appendix, we denote by q a rational or an imaginary quadratic integer with $|q| > 1$, and \mathbb{K} an imaginary quadratic number field including q . Note that \mathbb{K} must be of the form $\mathbb{K} = \mathbb{Q}(q)$ if q is an imaginary quadratic integer.
5. **Throughout** this section, we assume that d is odd.
6. We use the following notation **throughout**. We write $e(\alpha) = \exp(2\pi i\alpha)$, and denote the divisor function and Euler's totient function by $\tau(q)$ and $\varphi(q)$, respectively.

11 Thus

1. Thus, using (4.8) again, we obtain
 2. Thus it follows from (4.3) that
 3. Thus we see that
 4. Thus by Lemma 2.2 we have
 5. Thus it may be realized that to study the sum
 6. Thus, an integer a is a quadratic residue modulo p if and only if $(a, p) = 1$ and a has a square root modulo p .
 7. Thus, the derivation D_F on F is uniquely determined by the derivation D on R .
 8. Thus, there are only finitely many pairs of exponents (m, n) for which the Catalan equation is solvable.

12 for which

1. We shall first determine the set of primes p **for which** -1 is a quadratic residue.
 2. Let $E(K)$ be the set consisting of all elements $\alpha \in K$ **for which** the functional equation (2.1) has a polynomial solution.
 3. Hence, by (2.2), our main task is to determine the pairs (q, n) **for which** the functional equation (2.1) with $s = 2$, $P(z) = (z - q)^2$, and $u = q^n$ has a polynomial solution of degree $n \in \mathbb{N}$.
 4. Artin [5, p.84] states that there exist infinitely many primes **for which** a is a primitive root. Moreover, Artin has a conjectured density for the set of primes **for which** a is a primitive root.
 5. Thus, there are only finitely many pairs of exponents (m, n) for which the Catalan equation is solvable.

13 used

1. In the following sections we shall see how the Euler and Fermat theorems can be **used** to determine whether an integer is prime or composite, and how they are applied in cryptography.
2. This algebraic identity will be **used** in the next section to prove Mason's theorem.

3. We shall find asymptotic formulae for P and R by the same method **used** in the proof of Theorem 2.3.
4. The following result will be **used** in Section 3 to prove that the set of abundant numbers has an asymptotic density.
5. Finally, we state the following simple lemma, which will be **used** in the proof of Liouville's formula.
6. Hardy and Ramanujan [5] and Uspensky [6] independently discovered this result; their proofs **used** complex variables and modular functions.
7. In the last inequality we **used** the fact that $\sqrt{n} \geq \frac{c}{2}$. Therefore,
8. In 1918 Hardy and Ramanujan [5] published the asymptotic formula for the partition function. Uspensky [6] obtained the same result independently in 1920. Both papers **used** complex variables and modular functions to deduce the asymptotic estimate $p(n) \sim (4n\sqrt{3})^{-1} e^{c_0\sqrt{n}}$.
9. In this section we derive the two results about power series with nonnegative coefficients that were **used** to deduce Theorem 2.

14 so that

1. In this section we obtain an explicit formula for $R_6(n)$. The idea is to apply identity (4.3) to the monomials x^3y and xy^3 , and to manipulate the results **so that** we can find a function $\Phi(n)$ that satisfies the recursion formula

$$\sum_{|x| \leq \sqrt{n}} (n - 7x^2) \Phi(n - x^2) = 0.$$

2. Let p_n denote the n -th prime number, **so that** $p_1 = 2, p_2 = 3, \dots$

15 for all

for all positive integers k and real numbers t .

for all positive integers k and **for all** real numbers t . (**false**).

It follows that there exist numbers M and u_1 such that

It follows that there exist **a** numbers M and u_1 such that (**false**).

It follows that there exist **a** functions f and g such that (**false**).

16 let, and let

Let p be prime, **and let** $f(x)$ be a polynomial of degree n with integer coefficients and leading coefficient not divisible by p .

Let p be an odd prime, and let a be an integer not divisible by p .

Let G be an abelian group, written additively, and let A_1, \dots, A_k be subsets of G .

17 by

1. **By** continuity (By the continuity) of f, \dots
2. **By** assumption, \dots
3. **By** the above, \dots
4. **By** the induction hypothesis, \dots
5. **By** what we have already proved, $|f|$ must then be of the form \dots
6. Such a G_0 exists **by** (2) when $n \geq k$.
7. **By** considering translations, rotations, and reflections separately, it is not hard to see that \dots
8. Consider the class of finite graphs, **by which** we mean simple graphs, i.e., without loops or multiple edges.
9. To see this connection, we need to explain briefly the method **by which** universal minimal flows are calculated in [2].
10. If H is an induced subgraph of G , then μ_G induces μ_H **by** restriction.
11. The addition of a single hyperedge to G changes $N(G)$ **by** at most k .
12. **By** and large, we shall use the same notation as in \dots
13. term-**by**-term differentiation
14. Thus we need only alter our constants **by** a factor of 2 to deal with this case.
15. **By** quasi-equation we understand a sentence of the form \dots
16. **By** Lemma 1.2, there is a number $c_0 \geq 1$ such that if $0 < \delta < 1$ and $x \geq x_1(\delta) \geq 4$, then there exists an integer

$$n \in \left] x, e^{\frac{c_0}{\delta}} x \right] \quad (1)$$

with $|R(n)| \leq \delta n$.

18 each

1. Then F is bounded on **each** bounded set.
2. **Each** of these three integrals is finite.
3. These curves arise from..., and **each** consists of...
4. There remain four intervals of length $\frac{1}{4}$ **each**.
5. Here X assumes values $0, 1, \dots, 9$, **each** with probability $\frac{1}{10}$.
6. Then F_1, F_2, \dots, F_n vary **each** in the interval $[0, 1]$.
7. The first and third terms are **each** less than $\frac{\varepsilon}{3}$.
8. a progression **each** of whose terms can be written as....
9. These n disjoint boxes are translates of **each** other.
10. The two notions of rank are independent of **each** other.
11. For $j = 1, \dots, k$, we have

$$x_j \geq n^k \tag{2}$$

and so **each** interval $]x_j, x_{j-1}[$ contains a subinterval I_j such that

12. **Each** factor of the Euler product is nonzero, since ...
13. To **each** $b_1 \in B_1$ there exists at most one $b_2 \in B_2$ such that $b_1 + b_2 = n$.
14. For **each** such integer v_1 there is at most one integer v_2 that satisfies the linear equation (1.5).
15. Deduce that **if each** of the integers n_1 and n_2 can be represented as a sum of two squares, then their product $n_1 n_2$ is also a sum of two squares.
16. The function $f(x, y) = xy$ is odd in **each** of the variables x and y .
17. We write each partial fraction as a power series:

19 element

1. We shall call the elements of such a chain links.
2. Computing $f(y)$ can be done by enumerating $A(y)$ and testing each element for membership in C .
3. Two consecutive elements do not belong both to A or both to B .
4. a 3-element set

20 elementary

1. see also: easy, simple, straightforward, basic, primary
2. It is an **elementary** check that A is a vector space.
3. However, shortly after learning about Wiener's work, P. Levy found a more **elementary** argument.
4. By **elementary** algebra, we can show that....

21 too, see also: also, well, similarly, likewise.

1. In practice, D is usually **too** large a set to work with.
2. Consequently, A has two elements **too** many. [Or: A has two **too** many elements.]
3. There are other problems with this example which would hinder any attempt to follow the proof given here **too** closely.
4. We denote this, **too**, by Q .
5. Note that this **too** is best possible.
6. The inner sum is zero (and so **too** is $S(a, b)$).
7. If (1) and (2) hold, then so **too** does (3).

22 because, see also: as, for, since

1. However, this argument is fallacious, **because** as remarked after Lemma 3, K is discontinuous.
2. This is **because** the factor M satisfies condition (P).
3. Unfortunately, **because** of the possible presence of 'cusps', this need not be true.
4. **Because** of this, W is never long enough to cancel with M in the product ABC .
5. The statement $S(0)$ is true **because** if $1 \leq n < m$, then $n = a_0$ is the unique m -adic representation.
6. We see that this congruence does have a solution y_1 **because** $(v_1, p) = 1$.

23 being

1. Note that M **being** cyclic implies F is cyclic.
2. The probability of X **being** rational equals $\frac{1}{2}$.
3. This is exactly our definition of a weight **being** regulated.
4. We have to show that the property of there **being** x and y such that $x < y$ uniquely determines P up to isomorphism.
5. In addition to f **being** convex, we require that.....
6. Here J is defined to equal A_f , the function f **being** as in (3).
7., the constant C **being** independent of.....
8. The ideal is defined by $m = \dots$, it **being** understood that.....
9. But....., it **being** impossible to make A and B intersect.
10. The map F **being** continuous, we can assume that.....
11. Actually, S has the much stronger property of **being** convex.
12. This method has the disadvantage of not **being** intrinsic.

24 below

1. A brief sketch of the reasoning is given **below**.
2. By Remark 3 **below**,.....
3. The pressure increases are significantly **below** those in Table 2.
4. This proves that the dimension of S does not go **below** q .
5. a function bounded **below** (above) by 1.
6. As a first step we shall bound A **below**.

25 case

1. These are special **cases** of Waring's problem, one of the most famous problems in number theory.
2. **Proof.** We first consider the **case** where $m = p^t$ is a power of a prime p .
3. We first do the **case** $n = 1$.
4. This argument also settles the **case** of $K = \Gamma$.

5. We finish by mentioning that, suitably modified, the results of Section 2 apply to the *AP case*.
6. Note that (4) covers the other **cases**.
7. There are several **cases** to consider:
8. We close this article by addressing, in part, the **case** of what happens if we replace the map T by convolution.
9. There are quite a number of **cases**, but they can be described reasonably systematically.
10. The general **case** follows by changing x to $x - a$.
11. This abstract theory is not in any way more difficult than the special **case** of the real line.
12. Important **cases** are where $S = \dots$
13. This **case** arises when.....
14. Both **cases** can occur.
15. To deal with the zero characteristic **case**, let.....
16. Then either....., or..... In the latter (former) **case**,.....
17. In the **case** of finite additivity, we have.....
18. In the **case** of $n \geq 1$ (In **case** $n \geq 1$),..... [Better: If $n \geq 1$ then.....]
19. In the **case** where A is commutative, as it will be in most of this paper, we have.....
20. We shall assume that this is the **case**.
21. It cannot be that there exists $x \in \Omega$, for otherwise $\theta(\delta x) = \kappa E(\delta x) = 0$, which is not the **case**.
22. Unfortunately, this is rarely the **case**.
23. However, it need not be the **case** that $V > W$, as we shall see in the following example.
24. Such was the **case** in (8).
25. The L^2 theory has more symmetry than is the case in L^1 .
26. Note that some of the a_n may be repeated, in which case B has multiple zeros at those points.
27. Next we consider the general **case** where m has the standard factorization.
28. , and this is a special **case** of Theorem 2.2.

29. ... and so, by the **case** $k = 2$, there exists an integer x such that ...
30. In this **case**, the element a is called a generator of G .
31. Let us apply these remarks to the special **case** when $G = \dots$
32. In the **case** $k = 3$, we observe that 5 has order 2 modulo 8.
33. Since every odd prime p is congruent to 1, 3, 5, or 7 modulo 8, there are four **cases** to consider.
34. **Proof.** This is a special **case** of Lemma 1 in Section 2. \square
35. Many classical problems are special **cases** of this conjecture.
36. The Goldbach conjecture is the special **case** when N is even, $r = 1$ and $f_1(t) = g(t) = t$.
37. The proof is easier than the general **case**, and shows clearly the use of Dirichlet characters and Dirichlet L -functions.

26 **certain, certainly**

1. In this section we obtain upper bounds for **certain** linear and quadratic diophantine equations.
2. The spectrum of T was defined in [2] and identified with the spectrum of a **certain** algebra A_T .
3. It seems plausible that..... but we have been able to establish this only in **certain** cases.
4. under **certain** conditions
5. Let p be an odd prime number. Then p is **certainly** not a square. By Lemma 5, ...
6. We know the asymptotics of partition functions for certain sets of integers of zero density. For example, Hardy and Ramanujan [5] proved that if ...
7. We have **certain** views about the logic of the theory; we think that some theorems, as we say “lie deep” and others nearer to the surface.
8. This is **certainly** reasonable for Algorithm 3, given its simple loop structure.
9. Hence both decay exponentially as $x \rightarrow \infty$, therefore **certainly** remain bounded.
10. As Corollary 2 shows, it is **certainly** a question deserving further exploration.
11. Here, of course, the set A produced is rather thin and **certainly** nowhere near the densities we are looking for.

27 tend, see also: converge, approach

1. Clearly, F_n **tends** to zero as $n \rightarrow \infty$.
2. Observe that as I becomes smaller, $d(I, f)$ **tends** to 1.
3. This problem is still fundamentally unsolved, even though we know many beautiful results about the growth of $\pi(x)$ as x **tends** to infinity.

28 common

1. The functions f_i ($i = 1, \dots, n$) have no **common** zero in Ω .
2. Then F and G have a factor in **common**.
3. It has some basic properties in **common** with another most important class of functions, namely, the continuous ones.
4. Take g_1, \dots, g_n without **common** zero.
5. Denote by θ the angle at x that is **common** to these triangles.
6. Here we use an inductive procedure very **common** in geometric model theory.
7. Waring's problem for polynomials states that if the greatest **common** divisor of the set $A(f)$ is 1, then every sufficiently large integer can be written as the sum of a bounded number of elements of $A(f)$.
8. Let $\gcd(A)$ denote the greatest **common** divisor of the elements of the set A . If $\gcd(A) = d$, then every sum of elements of A is divisible by d . It follows that the set A is an asymptotic basis only if $\gcd(A) = 1$.
9. The greatest **common** divisor and the least **common** multiple of the integers a_1, \dots, a_k are denoted by (a_1, \dots, a_k) and $[a_1, \dots, a_k]$, respectively.

29 understand

1. When we talk of a complex measure, it is **understood** that $\mu(E)$ is a complex number.
2. The first equality is **understood** to mean that.....
3. The ideal is defined by $m = \dots$, it being **understood** that.....
4. To **understand** why, let us remember that.....
5. By quasi-equation we **understand** a sentence of the form.....

30 usual

1. This metric produces the **usual** topology of X .
2. A different notation is used because the **usual** tensor product symbol is reserved for the tensor product of A -bimodules.
3. The **usual** definition is more restrictive in that it requires that $a \in A$.
4. As **usual**, we can rephrase the above result as a uniqueness theorem. [Not: “As usually”]
5. with the usual modification for $p = \infty$
6. One **unusual** feature of the solution should be pointed out.

31 viewpoint

1. From the viewpoint of the Fox theorem, there is not an exact parallel between the odds and the evens.

32 why

1. To understand **why**, let us remember that.....
2. This is **why** no truncation is required here.
3. Now **why** can such objects be found?

33 definitely

1. The theorem is **definitely** false without the assumption that....., as an inspection of Example 2 shows.

34 multiple

1. Thus F is at most a multiple of G plus.....
2. The last term is bounded by a constant multiple of the norm of g .

35 **namely**

1. Here D_0 and D_1 are discs with the same centre, **namely** b .
2. The object of this paper is to obtain improvements in two cases, **namely** for forms of degree 7 and 11.
3. It has some basic properties in common with another most important class of functions, **namely**, the continuous ones.
4. There is another, entirely different, way to see that $A = B$. **Namely**, one can first show that.....
5. [Note the difference between **namely** and that is: while **namely** introduces specific or extra information, that is (or i.e.) introduces another way of putting what has already been said.]

36 **member**

1. Moreover, $\{x\}$ is the set whose only **member** is x .
2. Define $F : \omega \rightarrow \omega$ by setting $F(m)$ to be the largest **member** of the finite set X_m .
3. Examination of the left and right **members** of (1) shows that....

37 **object, see also: aim, purpose**

1. The **object** of this section is to classify all the indecomposable E -modules.
2. The **object** of this paper is to obtain improvements in two cases, namely for forms of degree 7 and 11.

38 **paragraph**

1. Let the notation be as in the preceding **paragraph**.
2. As the first **paragraph** of the proof will make clear, we can choose f in such a way that.....
3. (see the last **paragraph** but one of page 17)

39 **comparison**

1. Comparison of (2) and (3) gives..... [Or: A comparison]

40 considerable, see also: important, significant, substantial

1. However, they now differ by a **considerable** amount.
2. It is possible that the methods of this paper could be used to....., but there remain **considerable** obstacles to overcome.

41 contradiction, see also: contrary, otherwise

1. Assume, by way of **contradiction**, that there exists an unlimited positive integer m_0 such that $\delta^{s,\ell,m_0}(n) \simeq +\infty$.
2. To obtain a **contradiction**, we suppose that.....
3. Suppose, to derive a **contradiction**, that.....
4. Striving for a **contradiction**, suppose that.....
5. Aiming for a **contradiction**, suppose that.....
6. Suppose, towards a **contradiction** (for contradiction), that.....
7., which is a **contradiction**.
8. Now we have the required **contradiction** since.....
9. This leads to the **contradiction** that $0 < a < b = 0$.
10., in **contradiction** with Lemma 2.
11. This is a clear **contradiction** of the fact that.....

42 counterexample

1. In addition to illustrating how our formulas work in practice, it provides a **counterexample** to Brown's conjecture.
2. For a **counterexample**, consider $S = \dots$

43 proof

1. Here is a simple direct **proof**.
2. The others being obvious, only *(iv)* needs **proof**.
3. The major portion of one direction of the **proof** is contained in the previous proof.
4. The **proof** will only be indicated briefly.
5. We can assume that p is as close to q as is necessary for the following **proof** to work.
6. The **proof** follows very closely the **proof** of (2), except for the appearance of the factor x^2 .
7. The **proof** proper [= The actual proof] will consist of establishing the following statements in sequence.
8. The standard **proofs** proceed via the Cauchy formula.
9. An ingenious alternative **proof**, shorter but still complicated, can be found in [5].
10. Kim announces that (by a tedious **proof**) the upper bound can be reduced to 10.
11. The following has an almost identical **proof** to that of Lemma 2.
12. A close inspection of the **proof** reveals that.....
13. This finishes (completes) the **proof**.
14. The method of **proof** carries over to domains satisfying.....
15. This sort of **proof** will recur frequently in what follows.
16. We end this section by stating without **proof** an analogue of.....
17. It seems reasonable to expect that....., but we have no **proof** of this.
18. a laborious (complicated/routine/straightforward) **proof**

44 paper, see also: article

1. in **paper** [3] [Or: in the **paper** [3]; better: in [3]]
2. in a companion **paper** [4]
3. The aim of this **paper** is to bring together two areas in which.....
4. In the present **paper** we move outside the random walk case and treat time-inhomogeneous convolutions.

5. There are, however, a few important **papers** of which we were unaware until fairly recently.
6. These volumes bring together all of R. Bing's published mathematical **papers**.
7. In the present **paper** we apply nonstandard analysis in the field of number theory.

45 **propose, see also: suggest**

1. He was the first to **propose** a complete theory of triple intersections.
2. A model for analysing rank data obtained from several observers is **proposed**.
3. Unfortunately, the **proposed** model does not satisfy condition (5).

46 **page**

1. on **page** 13 [Not: "on the page 13"]
2. at the top of **page** 4
3. (see the last paragraph but one of **page** 24)
4. (see the **page** opposite)
5. Our method of proof will be an adaptation of the reasoning used on **pp.** 71 – 72 of [3].

47 **pair**

1. The ordered **pair** (a, b) can be chosen in 16 ways so as not to be a multiple of (c, d) .
2. There cannot be two edges between one **pair** of vertices.
3. Write out the integers from 1 to n . **Pair up** the first and the last, the second and next to last, etc.

48 **paralle**

1. The proof runs **parallel** to that of Lemma 2. [Not: "parallely"]
2. From the viewpoint of the Fox theorem, there is not an exact **parallel** between the odds and the evens.
3. The proof closely **parallels** that of Theorem 1.

49 abbreviate

1. We shall **abbreviate** the expression (1) to $F(k)$.
2. We **abbreviate** this as $f = g$ a.e.
3. Thus, in **abbreviated** notation,.....

50 abbreviation

1. Note that (3) is merely an abbreviation for the statement that.....

51 able, see also: can

1. We are able to estimate this part of (2.9) as $O\left(\frac{1}{\ln x}\right)$.
2. Using some facts about polynomial convexity, we are **able** to deduce.....
3. It seems plausible that..... but we have been **able** to establish this only in certain cases.
4. However, we have thus far been **unable** to find any magic squares with seven square entries.

52 about, [see also: roughly, approximately]

1. The Taylor expansion of f **about** (around) zero is.....
2. If s_0 lies below R_{-2} , then we can reflect **about** the real axis and appeal to the case just considered.
3. These slits are located on circles **about** the origin of radii r_k .
4. The diameter of F is **about** twice that of G .
5. Then $n(r)$ is **about** $k \cdot r^n$.
6. Let A denote the rectangle B rotated through $\frac{\pi}{6}$ in a clockwise direction **about** the vertex $(0, 1)$.
7. What would this imply **about** the original series?
8. What **about** the case where $q > 2$?
9. It is hoped that a deeper understanding of these residues will help establish new results **about** the distribution of modular symbols.

10. On the other hand, there is enormous ambiguity **about** the choice of M .
11. In this section we ask **about** the extent to which F is invertible.
12. Here the interesting questions are not **about** individual examples, but about the asymptotic behaviour of the set of examples as one or another of the invariants (such as the genus) goes to infinity.
13. However, as we are **about** to see, this complication is easily handled.
14. This brings **about** the natural question of whether or not there is any topology on the set of all possible itineraries

53 absence, [see also: lack]

1. The location of the zeros of a holomorphic function in a region Ω is subject to no restriction except the obvious one concerning the absence of limit points in Ω .
2. [Note the difference: absence = non-presence; lack = shortage of something desirable.]

54 absorb

The second term can be **absorbed** by the first.

55 abstract

1. It seems that the relations between these concepts emerge most clearly when the setting is quite **abstract**, and this (rather than a desire for mere generality) motivates our approach to the subject.
2. This **abstract** theory is not in any way more difficult than the special case of the real line.

56 abundance

[see also: wealth, variety, profusion, numerous] The monograph is illustrated with an **abundance** of figures and diagrams.

57 abuse

1. By **abuse** of notation, we continue to write f for f_1 .
2. We shall, by convenient **abuse** of notation, generally denote it by x_t whatever probability space it is defined on.
3. With the customary **abuse** of notation, the same symbol is used for.....

58 accessible

1. Thus the paper is intended to be **accessible** both to logicians and to topologists.
2. The present paper is motivated by the desire to make the subject as **accessible** as possible.

59 accordance, [see also: agreement]

Choose δ in accordance with Section 4.

60 according

1. [as X or Y ; to sth; to whether X or Y]
2. The solutions are f or g according as $t = 1$ or $t = 2$.
3. The curvature is positive, zero, or negative according to whether two geodesics initially perpendicular to a short geodesic arc through p converge, stay parallel, or diverge.
4. Then F can be decomposed according to the eigenspaces of P .
5. Choose S_k according to the following scheme.
6. The middle part of Table 2 compares the classification according to $\max a_i$, where only the longitudinal information is utilized, with those according to $\max b_{ik}$, where both longitudinal and survival information are used.

61 accordingly

[see also: respectively, suitably]

1. Accordingly Lemma 4.1 yields
...
2. We accordingly specify the relevant notation ...
3. The player has to decide which of the two strategies is better for him and act accordingly.

Write $A = BC$, factor $a = bc$ accordingly, and let.....

62 account

1. [of sth; see also: description]
2. A very readable account of the theory has been given in [4].
3. We are indebted here to Villani's account (see [2]) of a standard generalization of convex conjugacy.
4. For a recent account we refer to [4].
5. See the simplified account in [2, Section 4].
6. See [17] for a brief account of the results obtained. [Not: "the obtained results"]
7. The Markov chain z_k takes no account of how long the process stays in V .
8. On account of (5), we have.....
9. We must take account of the fact that A may have a substantial effect on the input length.
10. [for sth; see also: explain, justify, reason, represent]
11. This theorem accounts for the term "subharmonic". [= explains]
12. So all the terms of (2) are accounted for, and the theorem is proved.
13. He accounts for all the major achievements in topology over the last few years. [= He records]
14. Firms employing over 1000 people accounted for 50% of total employment. [= represented 50%]

63 achieve

1. [see also: attain, reach, take, gain]
2. Equality is **achieved** only for $a = 1$.
3. The function g **achieves** its maximum at $x = 5$.
4. Among all X with fixed L^2 norm, the extremal properties are **achieved** by multiples of U .
5. This **achieves** our objective of describing.....

64 achievement, [see also: result]

1. He accounts for all the major achievements in topology over the last few years.
2. Their remarkable achievement seemed to validate John's claim. However, it soon turned out that.....
3. a considerable (extraordinary/fine/important/impressive/outstanding/significant) achievement

65 acknowledge

1. We acknowledge a debt to the paper of Black [7].
2. This research was initiated when the first author was visiting the University of Alberta in the summer of 2008; the financial support and kind hospitality are gratefully acknowledged.
3. We also acknowledge useful discussions with J. Brown.
4. The author gratefully acknowledges the referee's helpful comments pertaining to the first draft of this paper.
5. [Do not write: "I acknowledge Dr. Brown for....." if you mean: I wish to thank Dr. Brown for.....]

66 action

1. All of the action in creating S_{i+1} takes place in the individual cells of type 2 or 3.
2. Away from critical points, the action of G is reminiscent of the action of a cyclic group of order d .
3. The goal of the present paper is to give a description of this kernel $T(G, H)$, valid for all G and H , in purely elementary terms, notably not using stable categories, nor representations, but essentially only the action of G by conjugation on the lattice of its p -subgroups.

67 actual

Actual construction of..... may be accomplished in a variety of ways. [= Real construction; do not use "actual" if you mean present or current.]

68 actually

1. [= in fact; despite what you may think; \neq at present; see also: fact, more].
2. The operator A is not merely symmetric, but actually selfadjoint.
3. Actually, Theorem 3 gives more, namely,.....
4. Actually, the proof gives an even more precise conclusion:.....
5. Although the definition may seem artificial, it is actually very much in the spirit of Darbo's old argument in [5].
6. Our present assumption implies that the last inequality in (8) must actually be an equality.
7. We then provide constructions to show that each of the cases listed can actually occur.
8. [Do not write: "Actually we prove (2) for $n = 1$ " if you just mean: Now we prove (2) for $n = 1$.]

69 a, an

1. If $p = 0$ then there are **an** additional m arcs.
2. This says that f is no longer than the supremum of the boundary values of G , **a** statement similar to (1).
3. Our present assumption implies that the last inequality in (8) must actually be **an** equality.
4. Hence all that we have to do is choose **an** x in X such that....
5. We conclude that there is **a** smallest integer n for which $f(n) = 0$.
6. Theorem 2 has **a** very important converse, the Radon-Nikodym theorem.
7. Some of the isomorphism classes above will have **a** rank of 2.

70 as well

1. To construct Padé approximations he **as well as** Thue uses hypergeometric polynomials, so this Padé approximation method is also called the hypergeometric method.
2. This method applies to all algebraic numbers, hence it is a very general theorem **as well as** Baker's theorem. The method is completely different from that of Baker.
3. The contents include several survey or half-survey articles (on prime numbers, divisor problems and Diophantine equations) **as well as** research papers on various aspects of analytic number theory such as additive problems, Diophantine approximations and the theory of zeta and L -functions.

4. Obviously the last formula holds for $x = 0$ **as well**.
5. The map ζ can be extended to the set K **as well as** in the usual case $s = 1$.

71 Assume

1. First we assume $r \geq 1$.
2. We may assume $r \geq 1$.
3. Assume that $x \in \mathbb{Q}$.
4. Throughout this paper we shall assume that ...
5. Shintani assumed that $\omega > 0$, but ...
6. In the rest of this section we assume $\frac{\pi}{2} \leq \theta \leq \pi$, and define
7. Let us assume (3). Then
8. We may assume that the number k is chosen to be the smallest positive number satisfying $x_k = x_0$.
9. We always assume the following condition:
10. In the statement of Theorem 4.2 in [2], we assume the condition (2.1).
11. Now, we assume that the Theorem is true for $n = k > 1$. That is,
12. Let us assume furthermore that all β_i ($i = 1, 2, \dots, k$) are rational. (not **rationals**)
13. Assume that h_0 is odd.

72 arrive

1. Setting $f = 0$, we **arrive** at a contradiction.

73 adapt

1. The proof of Theorem 5 is easily **adapted** to any open set.
2. The method of proof of Theorem B can be **adapted** to extend the right-to-left direction of Mostowski's result by showing that.....
3. This definition is well **adapted** for dealing with meromorphic functions.
4. The hypotheses of [4] are different, however, and do not seem to **adapt** easily to the time-inhomogeneous case.
5. Our method of proof will be an **adaptation** of the reasoning used on pp. 71 – 72 of [3].

74 above

1. The function F is bounded **above** (below) by 1.
2. By the **above**,.....
3. Let T be an isometric semigroup as **above**.
4. In the notation **above** (In the **above** notation),.....

75 add

1. The terms with $n > N$ **add** up to less than 2.
2. This interpretation does little, in sum, to **add** to our understanding of.....

76 addition

1. **Addition** of (2) and (3) gives.....
2. The set S is a semigroup with respect to coordinatewise **addition**.
3. If h is modified by the **addition** of a suitable constant, it follows that.....
4. The **addition** of a single hyperedge to G changes $N(G)$ by at most k .
5. In **addition** to illustrating how our formulas work in practice, it provides a counterexample to Brown's conjecture.
6. In **addition** to f being convex, we require that.....
7. In **addition** to a contribution to W_1 , there may also be one to W_2 .
8. Assume, in **addition** to the hypotheses of Exercise 4, that.....

77 additional

1. Now F has the additional property of being convex.
2. This solution has the additional advantage of being easily computable.
3. If $p = 0$ then there are an additional m arcs. [Note the article an.]

78 additionally, [see also: also, moreover]

Now F is additionally assumed to satisfy.....

79 address, [see also: deal, take up]

1. Strong compactness will be **addressed** in Section 3.
2. The main problems that we **address** are.....
3. **Addressing** this issue requires using the convergence properties of Fourier series.
4. We close this article by **addressing**, in part, the case of what happens if we replace the map T by convolution.

80 adhere

We **adhere** to the convention that $\frac{0}{0} = 0$.

81 adjoin

1. If we **adjoin** a third congruence to F , say $a = b$, we obtain.....
2. The extended real number system is R with two symbols, ∞ and $-\infty$, **adjoined**.

82 adjust, [see also: alter, adapt, change, modify]

1. In the latter case we may simply **adjust** F to equal 1 on the Borel set where it falls outside the specified interval.
2. The constants are so **adjusted** in (6) that (8) holds.

83 admit

1. The continuum Y is tree-like since it admits a map onto X .
2. This inequality admits of several interpretations.

84 adopt, [\neq adapt; see also: adhere, take]

1. We **adopt** throughout the convention that compact spaces are Hausdorff.
2. We **adopt** the convention that the first coordinate i increases as one goes downwards, and the second coordinate j increases as one goes from left to right.

3. To avoid undue repetition in the statements of our theorems, we **adopt** the following convention.
4. This is the point of view **adopted** in Section 3.
5. Furthermore, **adopting** this strategy considerably eases constructing a coding tree from a linear order.
6. We could have **adopted** an approach to proving Theorem 2 along a line of reasoning which bears greater resemblance to the treatment of the analogous result in Section 1.
7. Let us **adopt** the shorthand $F := FM_iN_i$.

85 **advance, [see also: development]**

1. The primary **advance** is to weaken the assumption that H is C^2 , used by previous authors, to the natural condition that H is C^1 .
2. Remarkable **advances** have been made recently in the understanding of....

86 **advantage**

1. One major **advantage** of.... is that.....
2. The **advantage** of using..... lies in the fact that.....
3. This solution has the additional **advantage** of being easily computable.
4. This approach fails to take **advantage** of the Gelfand topology on the character space.
5. We take **advantage** of this fact on several occasions, by not actually specifying the topology under consideration.
6. On the other hand, as yet, we have not taken **advantage** of the basic property enjoyed by S : it is a simplex.
7. a considerable (decisive/definite/obvious/main/significant) **advantage**

87 **affect, [see also: influence]**

1. Altering finitely many terms of the sequence u_n does not **affect** the validity of (9).
2. We show that one can drop an important hypothesis of the saddle point theorem without affecting the result.
3. How is the result **affected** if we assume merely that f is bounded?

4. If $a, b,$ and c are permuted cyclically, the left side of (2) is **unaffected**.
5. Properties involving topological centres are **unaffected** by a change to an equivalent weight.

88 affirmative

We give an **affirmative** answer to the question of [3].

89 afford, [see also: provide, furnish, supply, yield]

1. A counterexample is **afforded** by the Klein-Gordon equation.
2. We can now pose a problem whose solution will **afford** an illustration of how (5) can be used.
3. Having illustrated our method in Section 2, we can **afford** to be brief in our proof of Theorem 5.

90 aforementioned

1. Our first result generalizes (8) by exploiting some general facts seemingly overlooked by the **aforementioned** authors.
2. We underline that the **aforementioned** results in [1] all rely on the conformality of the underlying construction.

91 after

1. The proof of (8) will be given **after** we have proved that.....
2. We defer the proof of the “moreover” statement in Theorem 5 until **after** the proof of the lemma.
3. **After** making a linear transformation, we can assume.....
4. The desired conclusion follows **after** one divides by t and lets t tend to 0.
5. However, as pointed out right **after** (5),.....

92 again

1. Hence, by (7) **again**, we have.....
2. Finally, case (E) is completed by **again** invoking Theorem 1.
3. The operator H is **again** homogeneous.

93 against

No specific evidence against the conjecture has been produced yet.

94 agree

1. Our definition agrees with the one of [3].
2. The liftings on A and B agree on $A \cap B$, hence we can piece them together to obtain.....
3. Say the signatures agree in the j -th entry.

95 agreement, [see also: accordance]

1. This is in **agreement** with our previous notation.
2. With this **agreement**, it is clear that.....

96 aid

1. The solutions can be carried back to $H(V)$ with the **aid** of the mapping function ϕ .
2. We see with the **aid** of an integration by parts that.....
3. We now construct a group that will be of **aid** in determining the order of G .
4. We thank Professor Robin Harte for his substantial computational **aid**.
5. The first author would like to thank professor N. Azzouza for his **aid** on the numerical computation.

97 aim

1. See also: desire, end, object, objective, task, purpose, intention.
2. Our first aim is to study the ergodic properties of T .
3. Our aim here is to give some sort of “functorial” description of K in terms of G .
4. This connection has been exploited to construct various infinite families, with the aim of filling possible gaps. [Not: “with the aim to fill”]
5. In the remainder of this section, we study some properties of K , with the eventual aim (not realized yet) of describing K directly using G .
6. the broad (general/central/main/major/primary/limited/modest/underlying/original) aim
7. We aim to prove the following inequality:.....
8. These results therefore describe the very close connection between the method of encoding and the structures we are aiming to classify.
9. Aiming for a contradiction, suppose that.....

98 alas, [see also: unfortunately]

Having established (1), one might be tempted to try to extend this result to general p through the choice of a suitable ideal B . Alas, as we shall see now, this attempt is futile.

99 albeit, [= though]

1. However, we shall show in Section 3 that this simply results in Definition 3 again, **albeit** with complex weight.
2. It is proved in [1] (**albeit** with a slightly different formulation) that.....

100 algebra

By elementary **algebra**, we can show that.....

101 algorithm

1. It is obvious that the above theorem supplies an **algorithm** to effectively recognize whether SP is in A .
2. He used a new version of an **algorithm** for finding all normal subgroups of up to a given index in a finitely presented group.

102 all, [see also: any, each, every, whole, total]

1. Hence **all** that we have to do is choose an x in X such that.....
2. Thus, all that remains is to repeat the construction for f in place of g .
3. An examination of the argument just given reveals that this is all we have used.
4. All but a finite number of the G s are empty.
5. Note that any, but not all, of the sets αh^{-1} and βg^{-1} can be empty.
6. a manifold all of whose geodesics are closed [= a manifold whose geodesics are all closed]
7. Now E , F and G all extend to U .
8. They all have their supports in V .
9. They are all zero at p .
10. They should all be zero at p .
11. [Note the position of all in the last four examples: it is placed after the auxiliary verb; if there is no auxiliary, it is placed before the main verb, but if the verb is be, it is placed after it.]
12. This map extends to all of M .
13. These volumes bring together all of R . Bing's published mathematical papers.
14. If t does not appear in P at all, we can jump forward n places.
15. The last integral is over a horizontal line in P , and if this argument is correct at all, the integral will not depend on the particular line we happen to choose.
16. But $A_n z^n$ is much larger than the sum of all the other terms in the series $\sum A_k z^k$.
17. Thus A is the union of all the sets B_x .
18. the space of all continuous functions on X
19. the all-one sequence
20. Any vector with three or fewer 1s in the last twelve places has at least eight 1s in all.
21. The elements of G , numbering 122 in all, range from 9 to 2000.

103 allow

1. These theorems allow one to guess the Plancherel formula. [Or: allow us to guess; not: “allow to guess”]
2. As the space of Example 3 shows, complete regularity of X is not enough to allow us to do that.
3. This allows proving the representation formula without having to integrate over X .
4. This easily allows the cases $c = 1, 2, 4$ to be solved.
5. This allows the proof of the continuity of G to go through as before.
6. By allowing f to have both positive and negative coefficients, we obtain.....
7. It is therefore natural to allow (5) to fail when x is not a continuity point of F .
8. The limit always exists (we allow it to take the value ∞).
9. Lebesgue discovered that a satisfactory theory of integration results if the sets E_i are allowed to belong to a larger class of subsets of the line.
10. In [3] we only allowed weight functions that were C_1 .
11. It should be possible to enhance the above theorem further by allowing an arbitrary locally compact group L
12. Here we allow $a = 0$.
13. We deliberately allow that a given B may reappear in many different branches of the tree.
14. Case 3 is disallowed since it results in a disconnected curve on S , contradicting the tightness of P .

104 allude, [to sth; see also: mention, refer]

1. We now come to the theorem which was alluded to in the introduction of the present section.
2. One should remark that the ambiguity alluded to in Remark 3 disappears when talking about an affine field.

105 almost, [see also: nearly, practically]

1. It is almost as easy to find an element.....
2. Incidentally, the question of whether $K(E)$ is amenable for specific Banach spaces E seems to have received

almost no attention in the literature.

106 alone

1. Neither (1) nor (2) alone is sufficient for (3) to hold.
2. Now M does not consist of 0 alone.
3. Then F is a function of x alone.

107 along

1. This is derived in Section 3 along (together) with a new proof of Morgan's theorem.
2. The proof proceeds along the same lines as the proof of Theorem 5, but the details are more complicated.
3. For direct constructions along more classical lines, see [5].
4. Although these proofs run along similar lines, there are subtle adjustments necessary to fit the argument to each new situation.
5. Along the way, we come across some perhaps unexpected rigidity properties of familiar spaces.

108 already

1. This has already been proved in Section 4 [Not: "This has been proved already in Section 4."]
2. This idea is very little different from what can already be found in [2].
3. We put b in R unless a is already in.
4. In the physical context already referred to, K is the density of.... [Note the double r in referred.]
5. Inserting additional edges destroys no edges that were already present.

109 also, [see also: moreover, furthermore, likewise, too]

1. Hence f_n also converges to f .
2. We shall also leave to the reader the proof of (5).
3. Since R is a polynomial in x , so also is P .
4. The map G is not convex and also not C^1 . [Compare: The map F is not convex, and G is not convex either. Use either" when there is a similarity in the two negative statements.]

5. It is also not difficult to obtain the complete additivity of μ .

110 alter

1. We shall need ways of constructing new triangulations from old ones which alter the f -vector in a predictable fashion.
2. Altering finitely many terms of the sequence u_n does not affect the validity of (9).
3. The theorem implies that some finite subcollection of the f_i can be removed without altering the span.

111 alternate

1. The terms of the series (1) decrease in absolute value and their signs alternate.
2. Successive vertices on a path have alternating labels.

112 alternately

Every path on G passes through vertices of V and W alternately.

113 alternative

1. An alternative way to analyze S is to note that.....
2. Here is an alternative phrasing of part (1):.....
3. An ingenious alternative proof, shorter but still complicated, can be found in [12].

114 alternatively

Alternatively, it is straightforward to show directly that.....

115 although, [see also: though]

1. Although [1] deals mainly with the unit disc, most proofs are so constructed that they apply to more general situations.
2. Although these proofs run along similar lines, there are subtle adjustments necessary to fit the argument to each new situation.
3. Although the definition may seem artificial, it is actually very much in the spirit of Darbo's old argument in [5].

4. Now f is independent of the choice of γ (although the integral itself is not).
5. Thus, although we follow the general pattern of proof of Theorem 2, we must also introduce new ideas to deal with the lack of product structure.
6. Although standard, the notion of a virtual vector bundle is not particularly well known.

116 altogether, [see also: completely, total, whole]

1. However, we prefer to avoid this issue altogether by neglecting the contribution of B to S .
2. There are forty-three vertices altogether.

117 always

1. Since the lower bound (8) is always better than the lower bound (6), the worst lower bound (6) always holds.
2. ε always denotes a small positive constant
3. So we always have
4. We always assume the following condition:
5. By this proposition we see that Padé approximations to a given function exist always.
6. The problem is that, whatever the choice of F , there is always another function f such that.....
7. The induced topology is not compact, but we can always get it to be contained in a Bohr topology.
8. The vector field \mathbb{H} always points towards the higher A -level.

118 ambiguity

1. On the other hand, there is enormous ambiguity about the choice of M .
2. One should remark that the ambiguity alluded to in Remark 3 disappears when talking about an affine field.
3. When there is no ambiguity we drop the dependence on B and write just Y_T for $Y_{T,B}$.
4. This also resolves the ambiguity introduced earlier in choosing an order of the lifts of U .

119 among

1. [= amongst; see also: between, of, out of, include] Among the attempts made in this direction, the most notable ones were due to Jordan and Borel.
2. Among all X with fixed L^2 norm, the extremal properties are achieved by multiples of U .
3. If a_n is the largest among a_1, \dots, a_n , then.....
4. Our main results state in short that MEP characterizes type 2 spaces among reflexive Banach spaces.
5. The existence of a large class of measures, among them that of Lebesgue, will be established in Chapter 2.
6. There are several theorems for a number of other varieties. Among these are the Priestley duality theorem and.....
7. the number of solutions (x_1, \dots, x_n) in which there are fewer than r distinct values amongst the x_i
8. The next corollary shows among other things that..... [Not: “among others”]
9. Our result generalizes Urysohn’s extension theorems, among others. [= among other theorems]

120 amount

1. The quantities F and G differ by an arbitrarily small amount.
2. Thus θ will be less than π by an amount comparable to $a(s)$.
3. It is intuitively clear that the amount by which S_n exceeds zero should follow the exponential distribution.
4. It contributes half of the amount on the right hand side of (1).
5. [to sth; to doing sth; see also: total, add up] When $n = 0$, (7) just amounts to saying that.....
6. This just amounts to a choice of units.
7. Internet sales still amounted to only 3% of all retail sales in November.

121 analogous, [to sth]

1. **Problem 1.** Find a continued fraction expansion of $H(a, b, c; z)$ analogous to Gauss' continued fraction expansion to $G(a, b, c; 2)$.
2. These facts yield the following lemma which is analogous to Lemma 2 of the author's article [3]
3. The theory of..... is entirely (completely) analogous to.....
4. We shall also refer to a point as backward nonsingular, with the obvious analogous meaning.
5. Using (2) and following steps analogous to those above, we obtain.....

122 analogously, [to sth]

1. The lower limit is defined analogously: simply interchange sup and inf in (1).
2. The notion of backward complete is defined analogously by exchanging the roles of f and f^{-1} .
3. Analogously to Theorem 2, we may also characterize.....

123 analogue, [of sth; amer. analog]

1. This is an exact **analogue** of Theorem 1 for closed maps.
2. No **analogue** of such a metric appears to be available for Z .

124 analogy

1. Let us see what such a formula might look like, by **analogy** with Fourier series.
2. In **analogy** with (1) we have.....
3. There is a close **analogy** between.....
4. The **analogy** with statistical mechanics would suggest.....
5. Our presentation is therefore organized in such a way that the **analogies** between the concepts of topological space and continuous function, on the one hand, and of measurable space and measurable function, on the other, are strongly emphasized.

125 analyse, [see also: examine]

1. A model for analysing rank data obtained from several observers is proposed.
2. We are able to surmount this obstacle by analysing the rate of convergence.

126 analysis

1. [see also: exploration, investigation, study]
2. The only case requiring further analysis occurs when $f = 0$.
3. We now transfer the above analysis back to $M(A)$.
4. The analysis is similar to that of [3].
5. Analysis of the proofs of these previous results shows that.....
6. These results show that an analysis purely at the level of functions cannot be useful for describing.....
7. a careful (close/comprehensive/detailed/systematic/thorough) analysis

127 and

1. It seems that the relations between these concepts emerge most clearly when the setting is quite abstract, and this (rather than a desire for mere generality) motivates our approach to the subject.
2. It simplifies the argument, and causes no loss of generality, to assume.....
3. Thus the paper is intended to be accessible both to logicians and to topologists.
4. If one thinks of x, y as space variables and of z as time, then.....
5. He would like to express his appreciation to the faculty and staff of the Dartmouth mathematics department for their hospitality.
6. We need to check that F -derivatives behave in the way we expect with regard to sums, scalar multiples and products.
7. Thus A can be written as a sum of functions built up from B, C , and D . [Putting a comma before the and preceding the third object is standard in American usage.]

128 another

1. Another group of importance in physics is $SL_2(\mathbb{R})$.
2. In the next section we introduce yet another formulation of the problem.
3. It has some basic properties in common with another most important class of functions, namely, the continuous ones.
4. We have thus found another three solutions of (5). [= three more]

5. Here the interesting questions are not about individual examples, but about the asymptotic behaviour of the set of examples as one or another of the invariants (such as the genus) goes to infinity.
6. Then one Y_i can intersect another only in one point.
7. It is highly likely that if one of the X 's is exchanged for another, the inequality fails.

129 answer

1. [to sth; see also: explanation, solution]
2. An affirmative answer is given to the question of [3] whether.....
3. When A is commutative, the answer to both questions is "yes".
4. The algorithm returns 0 as its answer.
5. The answer depends on how broadly or narrowly the term "matrix method" is defined.
6. The answer is not known to us.
7. What is the answer if $a = 0$?
8. In the remainder of this section we shall be trying to answer the question:.....
9. This question was answered negatively in [5].
10. The two questions listed below remain unanswered.
11. As an application of Theorem A , in Section 2 we settle a question left unanswered in [3].

130 any

[see also: arbitrary, all, each, every, whatever, whichever]

1. By deleting the intervals containing x , if any, we obtain.....
2. There are few, if any, other significant classes of processes for which such precise information is available.
3. Let Q denote the set of positive definite forms
4. (including imprimitive ones, if there are any).
5. The preceding definitions can of course equally well be made with any field whatsoever in place of the complex field.
6. If K is now any compact subset of H , then there exists.....

7. Note that $F(t)$ may only be defined a.e.; choose any one determination in (7).
8. Note that any, but not all, of the sets α_{h-1} and β_{g-1} can be empty.
9. for any two triples [Not: “for every two triples”; “every” requires a singular noun.]

131 apart

[from sth; see also: besides, except, distinguish]

1. Apart from these two lemmas, we make no use of the results of [4].
2. Apart from being very involved, the proof requires the use of.....
3. There is a curve lying entirely in the open strip $0 < \gamma < 1$ apart from the endpoints such that.....
4. for no x apart from the unique solution of.....
5. [Note the difference between besides, except and apart from: besides usually indicates “adding” something, except “subtracts”, and apart from can be used in both senses; after no, nothing etc., all three can be used.]
6. Their centres are a distance at least N apart.
7. The m points x_1, \dots, x_m are regularly spaced t units apart.
8. What sets the case $n = 5$ apart is the fact that homotopic embeddings in a 5-manifold need not be isotopic.

132 apparatus

Keller, in his fundamental paper [7] concerning duality, develops an apparatus that allows him to obtain a very wide variety of duality theorems.

133 apparent

[see also: clear, evident, obvious, plain]

1. If one studies the proof of..... it is apparent that (2) is never used.
2. It is now apparent what the solution for K will be like:.....
3. This is usually called the area theorem, for reasons that will become apparent in the proof.
4. This formula makes it apparent that only the values $u(d)$ for positive d are relevant.

134 apparently

[see also: seemingly] Fox has apparently [= as one can see] overlooked the case of....

1. Note that the apparently [= seemingly] infinite product in the denominator is in fact finite.
2. The reader may wonder why we have apparently ignored the possibility of obtaining a better lower bound by considering.....

135 appeal, [see also: recourse]

1. Recently proofs have been constructed which make no **appeal** to integration.
2. In the preceding proof, the **appeal** to the dominated convergence theorem may seem to be illegitimate since.....
3. Through an **appeal** to (5.3) we have.....
4. [see also: invoke, refer] At this stage we **appeal** to Theorem 2 to deduce that.....
5. We can also **appeal** to Lemma 5 to see that the uniform continuity condition (5.3) is met.
6. If s_0 lies below R_{-2} , then we can reflect about the real axis and **appeal** to the case just considered.
7. One of the **appealing** aspects of the spectral set γ is that it readily lends itself to explicit computation.

136 appear

[see also: look, occur, seem, turn out, turn up, look]

1. However, no extension in this direction has appeared in the literature.
2. The statement does appear in [3] but there is a simple gap in the sketch of proof supplied.
3. It will eventually appear that the results are much more satisfactory than one might expect.
4. The zeros appear at intervals of $2m$.
5. Every prime in the factorization appears to an even power.
6. No analogue of such a metric appears to be available for Z .
7. Conditions relating to bounds on the eigenvalues appear to be rare in the literature.

8. At first glance, this appears to be a strange definition. [= seems to be]
9. Neighbourhoods of points in these spaces appear at first glance to have a nice regular structure, but upon closer scrutiny, one sees that many neighbourhoods contain collections of arcs hopelessly folded up.
10. This may appear rather wasteful, especially when n is close to m , but these terms only give a small contribution to our sum.
11. This conjecture also appears intractable at present.
12. It does not appear feasible to adapt the methods of this paper to.....

137 appearance

1. The only additional feature is the appearance of a factor of 2.
2. This convention simplifies the appearance of results such as the inversion formula.

138 applicability

The abstract theory gives us a tool of much wider applicability.

139 applicable [to sth]

1. We now provide a bound applicable to systems of.....
2. The hypothesis $n > 1$ ensures that Lemma 2 is applicable.

140 application

[see also: means, use, via]

1. Repeated application of (4) shows that.....
2. As an example of the application of Theorem 5, suppose.....
3. Even in the case $n = 2$, the application of Theorem 6 gives essentially nothing better than the inequality.....
4. Specifically, one might hope that a clever application of something like Choquet's theorem would yield the desired conclusion.
5. A drawback to P'olya trees, and perhaps the main reason why they have not seen much application within the Bayesian nonparametric literature, is that an arbitrary partition tree needs to be specified.
6. As an application, consider the Dirichlet problem $L_f = 0$.

7. We then show how this leads to stronger results in applications.
8. [Do not use “application” when you mean “map”: a map $f : X \rightarrow Y$ (not: “an application f ”).]

141 arbitrarily

1. The quantities F and G differ by an arbitrarily small amount. [Not: “arbitrary small”]
2.where C can be made arbitrarily small by taking.....
3. Runge’s theorem will now be used to prove that meromorphic functions can be constructed with arbitrarily preassigned poles.

142 arbitrary

[see also: all, each, every, whatever, whichever]

1. This enables us to define solution trajectories $x(t)$ for arbitrary t .
2. The theorem indicates that arbitrary multipliers are much harder to handle than those in $M(A)$.
3. One cannot in general let A be an arbitrary substructure of B here.
4. If X happens to be complete, we can define f on E in a perfectly arbitrary manner.

143 area, [see also: field]

1. I’m working in the area of Logic and Number Theory.
2. The region A has an area of $15 m^2$.
3. This is an interesting area for future research.
4. This is an area where there is currently a lot of activity.

144 argue [see also: assert, claim, reason]

1. To see that $A = B$ we argue as follows.
2. But if we argue as in (5), we run into the integral....., which is meaningless as it stands.
3. Arguing by duality we obtain.....
4. It might be argued that the h-principle gives the most natural approach to.....

145 argument, [see also: reasoning]

1. A similar argument holds for the other cases.
2. A deformation retract argument completes the proof.
3. In outline, the argument follows that of the single-valued setting, but there are several significant issues that must be addressed in the n -valued case.
4. This argument comes from [4].
5. This argument is invalid for several reasons.
6. However, this argument is fallacious, because as remarked after Lemma 3,.....
7. By an elementary argument,.....
8. This is handled by a direct case-by-case argument.
9. Following the argument in [3], set.....
10. The case $f = 1$ requires a different argument.
11. But the T_n need not be contractions in L_1 , which is the main obstruction to applying standard arguments for densities.
12. We give the argument when $I = R$.
13. Continuity then finishes off the argument.
14. This completes our argument for (1).
15. This assumption enables us to push through the same arguments.
16. It simplifies the argument, and causes no loss of generality, to assume.....

146 arise, [see also: emerge, occur, result]

1. In this paper we shall consider the general divisor problem which arises by raising the generating zeta-fuction $Z(s)$ to the k -th power, where the zeta-functions in question are the most general E. Landau's type ones that satisfy the functional equations with multiple gamma factors.
2., the last equality arising from (8).
3. This case arises when.....
4. The question arises whether.....
5. A further complication arises from 'BP', which works rather differently from the other labels.
6. If no confusion can arise, we write K for both the operator and its kernel.

147 arrow

Combining this with the attaching map defined above, we obtain the commutative diagram..... where surjectivity of the top-left arrow follows from the fact that.....

148 article, [see also: paper]

1. The first purpose of the present **article** is to eliminate the Riemann Hypothesis from the above result and prove the following result.
2. The second purpose of the present **article** is to consider a more general situation.
3. In the present **article**, we shall prove the following result which is a generalization of Theorem 1.
4. In this **article**, we shall prove that the above conjecture is correct as far as the asymptotic behavior as $\alpha \rightarrow \infty$ is concerned.
5. We close this **article** by addressing, in part, the case of what happens if we replace the map T by convolution.
6. We can now formulate the problem to which the rest of this **article** is dedicated.

149 artificial

Although the definition may seem **artificial**, it is actually very much in the spirit of Darbo's old argument in [5].

150 as

1. We can multiply two elements of E by concatenating paths, much as in the definition of the fundamental group.
2., where each function g is as specified hdescribedi above.
3. Actually, [3, Theorem 2] does not apply exactly as stated, but its proof does.
4. They were defined directly by Lax [2], essentially as we have defined them.
5. For $k = 2$ the count remains as is.
6. In the case where A is commutative, as it will be in most of this paper, we have.....
7. As a first step we identify the image of Δ .
8. Then F has T as its natural boundary.
9. The algorithm returns 0 as its answer.

10. Now X can be taken as coordinate variable on M .
11. If one thinks of x, y as space variables and of z as time, then.....
12. Then G is a group with composition as group operation.
13. We have $A \equiv B$ as right modules.
14. Then E is irreducible as an L -module.
15., as is easily verified.

151 ask

1. In this section we **ask** about the extent to which F is invertible.
2. This is the same as **asking** which row vectors in R have differing entries at positions i and j .
3. An obvious question to **ask** is whether the assertion of Theorem 1 continues to hold for.....

152 aspect

[see also: detail, feature, characteristic, ingredient, point]

1. One of the appealing **aspects** of the spectral set γ is that it readily lends itself to explicit computation.
2. I shall limit myself to three **aspects** of the subject.
3. We shall touch only a few **aspects** of the theory.

153 assert, [see also: say, state]

1. The spectral radius formula asserts that.....
2. Puiseux's theorem asserts the existence of.....
3. We also need the following technical lemma, which asserts the rarity of numbers with an inordinately large number of prime factors.
4. Here is a more explicit statement of what the theorem asserts.
5. To prove the asserted convergence result, first note that.....

154 **assertion, [see also: conclusion, statement]**

1. Now (2) is clearly equivalent to the assertion that.....
2., which proves the assertion. [Not: “the thesis”]
3. If we prove (8), the assertion follows.
4. The interest of the lemma is in the assertion that.....
5. Assertion (b) is known as the Radon Nikodym theorem.

155 **assess [see also: estimate]**

To assess the quality of this lower bound, we consider the following special case.

156 **assign, [see also: associate]**

1. The map f **assigns** to each x the unique solution of.....
2. As M is ordered, we have no difficulty in **assigning** a meaning to (a, b) .
3. A weighted graph is one in which each vertex is **assigned** an integer (called its weight).
4. Here the variable h is **assigned** degree 1.

157 **assume, [see also: suppose, presume]**

1. We can assume, by decreasing n if necessary, that.....
2. We may (and do) assume that.....
3. We tacitly assume that.....
4. It is assumed that.....
5. We follow Kato [3] in assuming that f is upper semicontinuous.
6. Here F is assumed to be open.
7., the limit being assumed to exist for every real x .
8. The assumed positivity of u_n is essential for these results.
9. The reader is assumed to be familiar with elementary K -theory.
10. Then X assumes values $0, 1, \dots, 9$, each with probability $1/10$.

158 assumption

[see also: condition, hypothesis, requirement]

1. We make two standing assumptions on the maps under consideration.
2. By the smoothness assumption on f ,.....
3. Because I is by assumption finite on A , it follows from (3) that.....
4. If the boundary is never hit then x_t is a Feller process under reasonable continuity assumptions.
5. We establish our results both unconditionally and on the assumption of the Riemann Hypothesis.
6. We note that the assumption of GCH is made for convenience and ease of presentation.
7. Then F is continuous at zero, contrary to assumption.
8. a basic (fundamental/implicit/tacit/underlying/reasonable/erroneous) assumption

159 at

1. At the same time, Selberg [2] also proved an unconditional result. He showed that, for almost all x ,
2. Hence we arrive at the conclusion
3. Values of Dirichlet series associated with modular forms at the points $s = \frac{1}{2}, 1$.
4. At the fourth comparison we have a mismatch.
5. At the suggestion of the referee, we consider some simple cases.
6. The match occurs at position 7 in T .
7. Now R is the localization of Q at a maximal ideal.
8. Next, F preserves angles at each point of U .
9. We may assume that this is the first point at which these two curves have met.
10. at the end of Section 2
11. Now F is defined to make G and H match up at the left end of I .
12. The zeros appear at intervals of $2m$.
13. We can make g Lipschitz at the price of weakening condition (i).
14. The two lines intersect at an angle of ninety degrees.

160 attach

The name of Harald Bohr is attached to bG in recognition of his work on almost periodic functions.

161 attain

[see also: achieve, reach, take]

1. Equality is attained only for $a = 1$.
2. The function g attains its maximum at $x = 5$.
3. Now (c) asserts only that the overall maximum of f on U is attained at some point of the boundary.

162 attention

1. It is a pleasure to thank R. Greenberg for bringing his criterion for..... to our attention, and for generously sharing his ideas about it.
2. We now turn our attention to.....
3. We can do this by restricting attention to.....
4. In this section I shall focus attention on.....
5. From now on we confine attention to R_2 .
6. Incidentally, the question of whether $K(E)$ is amenable for specific Banach spaces E seems to have received almost no attention in the literature.

163 auxiliary

In Section 2 the reader will be reminded of some important properties of Bernoulli numbers, and some auxiliary results will be quoted or derived.

164 available

1. No analogue of such a metric appears to be available for Z .
2. A further tool available is the following classical result of Chen.

165 aware, [see also: know]

1. Although the authors are not **aware** of any explicit reference except for the case $k = 2$ (see Schwarz [6]), a standard application of the circle method yields that for any....
2. At the time of writing [5], I was not **aware** of this reference.
3. One must also be **aware** that the curvature of M_i might not be bounded uniformly in i .
4. As far as we are **aware**, there is no proof in print.
5. In 1925 Franklin, **unaware** of Stackel's work, showed....

166 away, [see also: beyond, off, outside, distance]

1. Then F is smooth away hbounded awayi from zero.
2. Away from critical points, the action of G is reminiscent of the action of a cyclic group of order d .
3. If we keep x away from ∂D by restricting it to a compact set $K \subset D$, then.....
4. It follows that z is at least $2MN$ away from the left endpoint of I .

167 write, [see also: denote, symbol]

1. We write H for the value of G at zero.
2. We shall frequently write w.w. for 'weakly wandering'.
3., which we can write as $D_f = \dots$
4. Such cycles are said to be homologous (written $c \sim c_0$).
5. To simplify the writing, we take $a = 0$ and omit the subscripts a .
6. At the time of writing [5], I was not aware of this reference.
7. In the course of writing this paper we learned that P . Fox has simultaneously obtained results similar to ours in certain respects.
8. Important analytic differences appear when one writes down precisely what is meant by.....
9. It may be difficult to write down an explicit domain of F .
10. Note that (2) is simply (1) written out in detail.
11. By writing out the appropriate equations, we see that this is equivalent to.....
12. The lectures were written up by M . Stong.

168 well, [see also: also, too]

1. Obviously the last formula holds for $h = 0$ **as well**. Hence the latter assertion of the lemma follows from (3.4).
2. An improvement of this result may **well** be within reach, and we intend to return to this topic elsewhere.
3. By a **well-known** theorem of van der Corput, there exists a constant A such that the following inequalities are valid for all $X \geq 2$ and for all integers k with $1 \leq k \leq 5$.
4. For binary strings, the algorithm does not do quite **as well**.
5. But H itself can equally **well** be a member of S .
6. Our asymptotic results compare reasonably **well** with the numerical results reported in [8].
7. Since the integrands vanish at 0, we may **as well** assume that.....
8. Other types fit into this pattern **as well**.
9. Note that both sides of the inequality may **well** be infinite.
10. A cycle may very **well** be represented as a sum of paths that are not closed.
11. It may **well** be that no optimal time exists, as the following example shows.
12. Although standard, the notion of a virtual vector bundle is not particularly **well known**. variety
13. [see also: number, many, plentiful, several]
14. Actual construction of..... may be accomplished in a variety of ways.
15. The question of..... has been explored under a variety of conditions on A .
16. The approach in [4] provides a unified way of treating a wide variety of seemingly disparate examples.

169 various, [see also: different, several]

1. We shall be considering L on various function spaces.
2. We do not know how V depends on the various choices made.
3. Few of various existing proofs are constructive.
4. The author thanks the referee for recommending various improvements in exposition.

170 usually, [see also: normally]

1. It will **usually** be assumed that.....
2. The calculation of $M(f)$ is **usually** no harder than the calculation of $N(f)$.
3. In practice, D is **usually** too large a set to work with.
4. This topology is compact, but not **usually** Hausdorff, nor even T_1 .
5. In applications of Theorem 1, we are **usually** seeking a lower bound for $f(E)$.

171 upon

1. The second inequality follows upon considering R_i for $i > 0$.
2. We may assume, upon replacing F by F_1 , that.....
3. Upon combining the estimate for B with (5), we have now established the first conclusion of Theorem 8.
4. Adding E to both sides of (1), we can call upon (2) to obtain (3).

172 typically

1. A computational restraint is the algebraic number theory involved in finding these ranks, which will **typically** be more demanding than in our example of Section 1.
2. While nonparametric priors are **typically** difficult to manipulate, we believe the contrary is true for quantile pyramids.
3. Here one **typically** takes E to lie in the subspace H .

173 then

1. The complex case **then** follows from (a).
2. Continuity **then** finishes off the argument.
3. Theorem 3 may be interpreted as saying that $A = B$, but it must **then** be remembered that.....
4. **Then** G has 10 normal subgroups and as many non-normal ones.
5. If $p = 0$ **then** there are an additional m arcs. [Note the article an.]
6. If y is a solution, **then** ay also solves (3) for all a in B .

174 summarize

1. [\neq resume; see also: sum, summary]
2. We **summarize** some of its main properties, borrowing from the elegant discussion in Henson's article.
3. Theorems 2 and 3 may be **summarized** by saying that.....
4. **Summarizing**, whenever $n \geq 4$, we have shown that necessarily $p < 5$.

175 strategy

1. [see also: procedure, scheme, method, way]
2. Our basic strategy for proving (1) is different.
3. We start with a brief overview of our strategy.
4. Furthermore, adopting this strategy considerably eases constructing a coding tree from a linear order.
5. The condition..... can be improved by employing a strategy similar to that underlying the proof of Theorem 2.
6. The strategy is much the same as for the proof of Theorem 2.
7. a basic (broad/general/overall/viable) strategy

176 similarly, [see also: likewise]

1. Similarly, $b_1 = b_2$.
2. Similarly,
$$2^5 \equiv 1 \pmod{31}, \tag{3}$$
3. Similarly, 3 is a primitive root modulo 10, since $\varphi(10) = 4$.
4. Therefore, $a \notin \mathbb{Z}$ and, similarly, $b \notin \mathbb{Z}$.
5. Then F is similarly obtained from G .
6. Similarly to [4], we first consider the nondegenerate case. [Or: Just (Much) as in [4]; not: "Similarly as in [4]"]
7. Here we consider a dual variational formulation which can be derived similarly to that for the sandpile model.
8. Each $A(n)$ corresponds to an element $A_0(n)$ in V , and similarly for $B(n)$.
9. Similarly, if $f(t)$ is decreasing on the interval $[n, n + 1]$, then

177 similar

1. **Proof.** This is similar to the proof of (24) in Theorem 4.
2. [to sth; see also: like, reminiscent, resemble]
3. It is in all respects similar to matrix multiplication. [Not: “similar as”]
4. In the course of writing this paper we learned that P. Fox has simultaneously obtained results similar to ours in certain respects.
5. The proof is similar in spirit to that of [8].
6. Analysis similar to that in Section 2 shows that.....
7. Similar arguments to those above show that.....
8. This says that I is no longer than the supremum of the boundary values of G , a statement similar to (1).
9. A similar result holds for.....
10. Assume $a_1 < a_2$, the argument being similar in all the other cases.
11. in a similar fashion
12. in an exactly similar way similarity [see also: resemblance] This shape bears a striking similarity to that of.....
13. Note the similarity with Lusin’s theorem.

178 second [see also: other]

1. the second largest element
2. the second last row = the last but one row
3. the second author = the second-named author
4. To guarantee $Q(3)$, we use M_A a second time to control the perfect sets.
5. A second technique for creating new triangulations out of old ones is central retriangulation.
6. The first statement is obvious, since every flow contains a minimal subflow. For the second, it is enough to show that if.....

179 run

1. The rest of the proof runs as before.
2. Although these proofs run along similar lines, there are subtle adjustments necessary to fit the argument to each new situation.
3. Let A_0 be A run backwards.
4. As t runs from 0 to 1, the point $f(t)$ runs through the interval $[a, b]$.
5. The problem one runs into, however, is that f need not be smooth.
6. But this obvious attack runs into a serious difficulty.
7. But if we argue as in (5), we run into the integral....., which is meaningless as it stands.
8.where E runs over (runs through) the family B .

180 roughly [see also: approximately, about]

1. Roughly speaking, we shall produce a synthesis of index theory with Fourier analysis.
2. This says (roughly speaking) that the real part of g is.....
3. Roughly 0.7 comparisons were done for each character.

181 resolve, [see also: solve]

1. However, to our knowledge this is not fully resolved.
2. This also resolves the ambiguity introduced earlier in choosing an order of the lifts of U .
3. The case where $p > 1$ remains unresolved.

182 rephrase

1. [see also: reformulate, restate, rewrite]
2. As usual, we can rephrase the above result as a uniqueness theorem.
3. To deduce Theorem 1 from Proposition 2, we use a result of Silberg, which we will rephrase slightly for our purposes.
4. Rephrased in the language of [11], Proposition 2 says that.....
5. A rephrasing of the definition is that.....

183 repeatedly

1. [see also: frequently, often]
2. This property will be used **repeatedly** hereafter.
3. Let $m \geq 1$. If $d(n) \geq 4$, then by applying (3.2) **repeatedly** we obtain

$$\delta^{1,1,m}(n) \dots \leq \delta^{1,1,2}(n) \leq \delta^{1,1,1}(n) = d(n).$$

184 relatively, [= to a certain degree]

1. **Theorem.** The probability that two positive integers are **relatively** prime is $\frac{6}{\pi^2}$.
2. Therefore, the frequency of **relatively** prime pairs of positive integers not exceeding N is
3. Let m and n be **relatively** prime integers,
4. By relatively straightforward means one can show that.....
5. [Do not write: “the complement of A relatively to B ” if you mean: then complement of A relative to B .] relax [see also: weaken] The idea is to relax the constraint of being a weight function in Theorem 3.
6. The assumption that the test statistics are identically distributed can be relaxed without much difficulty.

185 relation

1. [of sth to sth; between sth; see also: connection, link, relationship]
2. But there is a much more important relation between equivalent regions:.....
3. By carefully examining the relations between the quantities U_i , we see that.....
4. There is a fourth notion of phantom map which bears the same relation to the third definition as the first does to the second.
5. What relation exists between f and g ?

186 recall

1. We recall what this means. [Not: “We remind”]
2. Recall the definition of T from Section 3.

187 random

1. The random variable X has the Poisson distribution with mean v .
2. In this and the other theorems of this section, the X_n are any independent random variables with a common distribution.
3. Let T_1, \dots, T_r be i.i.d. uniform $[0, 1]$ random variables conditioned to sum to 0 modulo 1.
4. To calculate (2), it helps to visualize the S_n as the successive positions in a random walk.
5. The proof shows that if the points are drawn at random from the uniform distribution, most choices satisfy the required bound.
6. Let A and B be its two parts, named in random order.

188 put, [see also: insert, plug, set]

1. Put a subset U of $j(X)$ in T if its inverse image under j is an open subset of X .
2. Put a taxicab metric on S_k .
3. The map F can be put into this form by setting.....
4. Put this way, the question is not precise enough.
5. This puts a completely different perspective on Fox's results.
6. This puts us in a position to apply Lemma 2 to deduce that.....
7. We put off discussing this problem to Section 5.
8. Putting these results together, we obtain the following general statement.....

189 occurrence

1. Replace each **occurrence** of b by c .
2. Throughout, C denotes a positive constant, not necessarily the same at each **occurrence**.

190 occurrence

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191 occurrence

1. Replace each **occurrence** of b by c .
2. Throughout, C denotes a positive constant, not necessarily the same at each **occurrence**.

192 obey, [see also: satisfy]

1. We first check that t_0 obeys the condition for $f(t)$.
2. One easily checks that whenever a sequence a_n obeys the uniform bound $a_n < C$, one has.....
3. It follows that any itinerary that obeys these four rules corresponds to a point in B .

193 now

1. The result will **now** be derived computationally.
2. Morera's theorem shows **now** that f is holomorphic.
3. **Now** that we have the above claim, we can select.....
4. **Now** choose a cycle c in M as in Theorem 2.

194 nowhere, [see also: far]

1. Here, of course, the set A produced is rather thin and certainly **nowhere** near the densities we are looking for. [The phrase nowhere near indicates that A is in fact far from reaching our expectations.]
2. a **nowhere** vanishing vector field

195 note, [see also: notice, observe, see, remark]

1. Note that (3) is merely an abbreviation for the statement that.....
2. Perhaps it is appropriate at this point to note that a representing measure is countably additive if and only if.....
3. Part (b) follows from (a) on noting that $A = B$ under the conditions stated.
4. Before going to the proof, it is worth noting that.....
5., as noted has was noted in Section 2. [Not: "as it was noted"]
6. As noted before, there exists N homeomorphic to P such that.....

7. It should be noted that.....

8. With the exception noted below, we follow Stanley's presentation [3, Sec. 2].

196 merely, [see also: just, only]

1. It turns out that A is not **merely** symmetric, but actually selfadjoint.

2. But (3) is **merely** an abbreviation for the statement that.....

3. Here (6) **merely** means that.....

4. How is the result affected if we assume **merely** that f is bounded?

197 member, [see also: element, side]

1. Moreover, $\{x\}$ is the set whose only member is x .

2. Define $F : \omega \rightarrow \omega$ by setting $F(m)$ to be the largest member of the finite set X_m .

3. Examination of the left and right members of (1) shows that.....

198 membership, [in/of sth]

1. Computing $f(y)$ can be done by enumerating $A(y)$ and testing each element for **membership** in \mathbb{C} .

2. Thus W satisfies two of the four requirements for **membership** in \mathbb{Z} .

199 mere, [see also: just, only]

1. The quantity A was greater by a **mere** 20%.

2. However, this equality turned out to be a **mere** coincidence.

3. This result shows that the **mere** existence of a nontrivial automorphism j of M produces the cut $I(j)$ of M that satisfies (2).

200 meaningless, [see also: meaningful]

But if we argue as in (5), we run into the integral....., which is meaningless as it stands.

201 means

1. [see also: application, device, use, via]
2. This provides an effective means for computing the index.
3. By relatively straightforward means one can show that.....
4. Then F and G are homotopic by means of a homotopy H such that.....
5. It is easy to see, by means of an example, that.....
6. Find integral formulas by means of which the coefficients c_n can be computed from f .
7. The difficulty is that it is by no means clear what one should mean by a normal family.

202 measure

1. Here dx stands for Lebesgue measure. [Or: the Lebesgue measure]
2. Each set A carries a product measure.
3. The aim of this article is to study the relationship between the size of A , as measured by its diameter, and the extent to which A fails to be convex.

203 meet

1. [see also: intersect, encounter, come across, run into, satisfy]
2. The sets A and B meet in two points. [= Their intersection is a two-point set]
3. We may assume that this is the first point at which these two curves have met.
4. Each component which meets X lies entirely within Y .
5. The remaining requirements for a type F map are also met.
6. We can also appeal to Lemma 5 to see that the uniform continuity condition (5.3) is met.

204 mean

1. Indeed, N is a Gaussian random variable with mean 0 and variance g .
2. [see also: signify, indicate, convey, suggest] We partially order M by declaring $X < Y$ to mean that.....
3. Here (1) can be interpreted to mean that.....
4. Here ‘essentially’ means ‘up to a zero set’.

5. The first equality is understood to mean that.....
6. In Chapter 5, we shall explain what it means for a subset V of A to be determining for the centre of X .
7. Note that (A) means precisely that condition (B) is not satisfied.
8. Important analytic differences appear when one writes down precisely what is meant by.....
9. Then (6) merely means that.....
10. The difficulty is that it is by no means clear what one should mean by a normal family. meaning [see also: sense] We shall also refer to a point as backward nonsingular, with the obvious analogous meaning.
11. As M is ordered, we have no difficulty in assigning a meaning to (a, b) .

205 meaningful

[see also: justify, legitimate, meaningless] This shows that the sequence (1) is bounded below, and so the definition of $L(f)$ is **meaningful**..

206 mapping

1. [see also: map, transformation, function]
2. We regard (1) as a **mapping** of S_2 into S_2 , with the obvious conventions concerning the point ∞ .
3. It is important to pay attention to the ranges of the **mappings** involved when trying to define.....
4. The **mapping** f leaves the origin fixed.
5. Bruck's theorem on common fixed points for commuting nonexpansive **mappings** is then brought into play by noting that.....

207 manner [see also: fashion, way, method]

1. Theorem 2, at the end of Section 2, was not originally obtained in the manner indicated there.
2. If our measure happens to be complete, we can define f on E in a perfectly arbitrary manner.
3. It is an easy matter to use Theorem 10 to construct all manner of interesting Peano continua [= continua of different kinds]

208 many

1. [see also: abound, number, numerous, profusion, several, variety, abundance]
2. Many of them were already known to Gauss.
3. The proof makes use of many of the ideas of the general case, but in a simpler setting.
4. Thus G has 10 normal subgroups and as many non-normal ones.
5. Consequently, H is a free R -module on as many generators as there are path components of X .
6. Therefore, A has two elements too many. [Or: A has two too many elements.]
7. Then A has three times as many elements as B has.
8. It meets only countably many of the Y_i .
9. a sequence with only finitely many terms nonzero
10. To compute how many such solutions there are, observe.....
11. How many of them are convex?
12. How many such expressions are there?
13. How many entries are there in this section?
14. How many multiplications are done on average?
15. How many zeros can f have in the disc D ?

209 lower

1. [see also: decrease, diminish, reduce, cut down, limit]
2. The importance of these examples lay not only in lowering the dimension of known counterexamples, but also in..... [Note that the past tense of lie is lay, not "lied".]
3. The lower limit is defined analogously: simply interchange sup and inf in (1).
4. The reader may wonder why we have apparently ignored the possibility of obtaining a better lower bound by considering.....
5. a path obtained by going from A to B along the lower half of the circle
6. a lower semicontinuous function

210 likely, [see also: possible, plausible]

1. All inputs of size n are equally likely to occur.
2. It seems likely that the arguments would be much more involved.
3. It is highly likely that if one of the X 's is exchanged for another, the inequality fails.
4. We expect that this is likely to hold for all others, but cannot prove this as yet.
5. A complete explication of the Fox spaces is warranted, as it will likely reveal further clues to the differences between the parabolic and hyperbolic theories.
6. This change is unlikely to affect the solution.
7. It is unlikely that the disturbances will eventually disappear.

211 likewise

1. [see also: similarly, also, moreover]
2. **Likewise**, if A does not span $C(I)$, removal of any of its elements will diminish the span.

212 limit

1. The limit $\lim_{x \rightarrow 0} f(x)$ exists. [Not: "There exists a limit $\lim_{x \rightarrow 0} f(x)$."]]
2. Now (1) follows after passage to the limit as $n \rightarrow \infty$.
3. [see also: confine, restrict] I shall limit myself to three aspects of the subject.

213 lend

1. One of the appealing aspects of the spectral set γ is that it readily lends itself to explicit computation.
2. This lends precision to an old assertion of Dini:.....

214 length, see also: detail, depth, expand

1. Pick the first arc of length 1 in this sequence.
2. The interval J has length 2^k .
3. It can be shown that the nearest point projection p reduces length by a factor of $\cos \alpha$.

4. This subject is treated at length in Section 2.
5. We shall discuss this again at somewhat greater length in Section 2.1.

215 legitimate

1. [see also: justify, meaningful]
2. Theorem 2 makes it legitimate to apply integration by parts.
3. The definition is legitimate, because.....
4. The interchange in the order of integration was legitimate, since.....
5. In the preceding proof, the appeal to the dominated convergence theorem may seem to be illegitimate, since.....

216 lemma

1. We shall prove this theorem shortly, but first we need a key **lemma**.
2. The proof will be divided into a sequence of **lemmas**.
3. We now prove a **lemma** which is interesting in its own right.
4. The interest of the **lemma** is in the assertion that.....
5. We defer the proof of the “moreover” statement in Theorem 5 until after the proof of the **lemma**.
6. With **Lemma** 4 in (at) hand, we can finally define E to be equal to $P(m)/H$.
7., from which it is an easy step, via **Lemma** 1, to the conclusion that.....
8. The following **lemma** is the key to extending Wagner’s results.
9. The following **lemma**, crucial to Theorem 2, is also implicit in [4].
10. Note that this **lemma** does not give a simple criterion for deciding whether a given topology is indeed of the form T_f .
11. At first glance **Lemma** 2 seems to yield four possible outcomes.
12. The final **lemma** is due to F. Black and is included with his kind permission.

217 inverse

1. [see also: converse, opposite, reverse]
2. Then F is the homeomorphism $X \rightarrow Y$ inverse to G .
3. No x has more than one inverse.
4. We use upper case letters to represent inverses of generators.
5. the inverse image = the preimage

218 investigate

1. [see also: examine, explore, study, test] In this section we investigate under what conditions the converse holds.
2. The above construction suggests investigating the solutions of.....
3. The situations with domains other than sectors remain to be investigated.

219 investigation

1. [see also: study, research, analysis, exploration]
2. We shall pursue our investigation of conservation laws in Section 5.
3. Theorem B is the main result of our investigation of stable ergodicity which we made in collaboration with A . Burks.
4. This paper, for the most part, continues this line of investigation.
5. An original impulse for this investigation came from the study of.....
6. The results have been encouraging enough to merit further investigation.
7. So one is naturally led to an investigation of.....
8. a careful (close/detailed/extensive) investigation

220 invariant

1. Clearly, F leaves the subspace M invariant.
2. Hence F is invariant under φ .
3. Thus F is G -invariant invariant under the action of G_i .
4. This set is clearly translation invariant.

5. Here the interesting questions are not about individual examples, but about the asymptotic behaviour of the set of examples as one or another of the invariants (such as the genus) goes to infinity.

221 introduce, [see also: set up]

1. We find ourselves forced to introduce an extra assumption.
2. In order to state these conditions succinctly, we introduce the following terminology.
3. We now introduce the algebras we shall be concerned with.
4. This motivated the second author to introduce the notion of.....
5. More specialized notions from Banach space theory will be introduced as needed.

222 introduction

1. The projection technique requires the introduction of an appropriate homomorphism.
2. This suggests the introduction of the differential operator $A = \dots$
3. We now come to the theorem which was alluded to in the introduction of the present chapter.

223 infer

1. [see also: conclude, deduce, follow] From (5) we infer that.....
2. We have shown that....., whence it is readily inferred that.....
3. It can be inferred from known results that these series at best converge conditionally in L^p .
4. Now apply Theorem 4.1 of [3] to infer the strong convergence at $e^{i\theta}$ of the Fourier series for Φ .

224 identical, [see also: same]

1. The obvious rearrangement reveals the right side to be identical with (8).
2. The following has an almost identical proof to that of Lemma 2.

225 identically

the identically zero map

226 identify

1. As a first step we identify the image of Δ .
2. Letting $m \rightarrow \infty$ identifies this limit as H .
3. Using the standard inner product we can identify H with H^* .
4. The tangent space to N at x is identified with M via left translation.
5. We henceforth identify $S_C(K \times K)$ with a^* -subalgebra of $L^\infty(X \times X)$.
6. The resulting metric space consists precisely of the Lebesgue integrable functions, provided we identify any two that are equal almost everywhere

227 however

1. [see also: but, though, nevertheless, matter]
2. In this section, however, we shall not use it explicitly.
3. That approach was used earlier in [2]. There, however, it was applied in simply connected regions only.
4. [Avoid using “however” as a simple substitute for but.]
5. This implies that however we choose the points y_i , the intersection point will be their limit point. [= no matter how we choose]
6. However small a neighbourhood of x we take, the image will be.....
7. Then $M_n(x) = 0$ for every x , however large.

228 hold

1. [see also: apply, true, valid, force]
2. All our estimates hold without this restriction.
3. The desired inequality holds trivially whenever $A > 0$.
4. In this section we investigate under what conditions the converse holds.
5. An obvious question to ask is whether the assertion of Theorem 1 continues to hold for.....
6. Equality holds hoccursi in (9) if.....
7. This attempt is doomed because the homogeneity condition fails to hold. [= The attempt is certain to fail]

8. The first author holds a Rockefeller Foundation fellowship.
9. He held the Courant Chair at New York University for three years before his retiring.
10. In the year 2000 (In 2000), two important number theory conferences were held at Princeton University.

229 hope

1. This leaves the **hope** that a ratio theorem may persist in a more general setting.
2. [see also: expect, expectation] A simple argument shows that we cannot **hope** to have $Df = 0$.
3. We cannot **hope** to say anything about the structure of each isotropy factor as a system in its own right.
4. Almost everywhere convergence is the best we can **hope** for.
5. Specifically, one might **hope** that a clever application of something like Choquet's theorem would yield the desired conclusion.
6. If nothing else, I **hope** to convince my readers that Segal's theorem deserves recognition as a profound contribution to Gaussian analysis.
7. It is **hoped** that a deeper understanding of these residues will help establish new results about the distribution of modular symbols.

230 hard

1. [see also: difficult, complex, complicated, involved, intricate]
2. This is the hard part of Jones's theorem.
3. The theorem indicates that arbitrary multipliers are much harder to handle than those in $M(A)$.
4. The calculation of $M(f)$ is usually no harder than the calculation of $N(f)$.
5. The cases $p = 1$ and $p = 2$ will be the ones of interest to us, but the general case is no harder to prove.
6. This makes G not hard to describe by generators and relations.

231 growth

1. The structure of a Banach algebra is frequently reflected in the growth properties of its analytic semigroups.
2. We give a fairly simple description of a wide class of averaging operators for which this rate of growth can be seen to be necessary.
3. the growth rate of V_n as $n \rightarrow \infty$
4. a function of moderate growth

232 guarantee, [see also: ensure]

1. Analyticity of the geodesic flow is no **guarantee** of observe how the completeness of L^2 was used to **guarantee** the existence of f .
2. However, (5) is sufficient to **guarantee** invertibility in A .
3. We are **guaranteed** only one dense product for each k .
4. This **guarantees** that f satisfies all our requirements.
5., which, by another theorem of Kimney's, is more than enough to **guarantee** that P gives A outer measure 1.

233 great

1. [see also: large, more, profound] Then F is 3 greater than G .
2. Thus F is not hnoi greater than G .
3. Consequently, F is greater by a half.
4. However, F can be as great as 16.
5. One should take great care with.....
6. In his Stony Brook lectures, he laid great emphasis on the use of.....
7. Another topic of great interest is how much of adjunction theory holds for ample vector bundles.
8. We could have adopted an approach to proving Theorem 2 along a line of reasoning which bears greater resemblance to the treatment of the analogous result in Section 1.
9. To show the greater simplicity of our method over Brown's, let us.....
10. It seems preferable, for clarity's sake, not to present the construction at the outset in the greatest generality possible.

234 go, [see also: stroke]

1. Its role is to rule out having two or more consecutive P-moves (on the grounds that they can be performed in one go).
2. [see also: continue, proceed, pursue, turn] a path obtained by going from A to B along the lower half of the circle
3. Here the interesting questions are not about individual examples, but about the asymptotic behaviour of the set of examples as one or another of the invariants (such as the genus) goes to infinity.
4. This proves that the dimension of S does not go below q .
5. We adopt the convention that the first coordinate i increases as one goes downwards, and the second coordinate j increases as one goes from left to right.
6. Some members go into more than one V_k .
7. To go into this in detail would take us too far afield.
8. We now go through the clauses of Definition 3.
9. Before going to the proof, it is worth noting that....
10. This idea goes back at least as far as [3].
11. This argument goes back to Banach.
12. Many of these results are known, and indeed they go back to the seminal paper of Dixmier [11] of 1951.
13. Going back to the existential step of the proof, suppose that.....
14. Before we go on, we need a few facts about the spaces L_p .
15. The equation $P_K = 0$ then goes over to $Q_K = 0$.
16. The rest of the proof goes through as for Corollary 2, with hardly any changes.
17. There are kneading sequences for which the arguments of Section 4 go through routinely.
18. This allows the proof of the continuity of G to **go** through as before.

235 give

1. [see also: provide, offer, supply, furnish, afford, define, yield, imply, present, produce]
2. This gives (1) and shows that.....
3. However, a slight strengthening of the hypotheses does give us a regular measure.
4. We now give some applications of Theorem 3.
5. We give X the topology of uniform convergence on compact subsets of I .
6. The action of G is given by $g_f = \dots$
7. The argument just given shows that.....
8. We first show that f satisfies the characterization given.
9. Note that E can be given a complex structure by setting.....
10. Suppose we are given an f of the form $f = \dots$
11. Given $\delta > 0$, we can find ε such that.....
12. This is certainly reasonable for Algorithm 3, given its simple loop structure.
13. He used a new version of an algorithm for finding all normal subgroups of up to a given index in a finitely presented group.
14. A function given on G gives rise to an invariant function on G_0 .

236 glance

1. [see also: look, sight] At first glance Lemma 2 seems to yield four possible outcomes.
2. Neighbourhoods of points in these spaces appear at first glance to have a nice regular structure, but upon closer scrutiny, one sees that many neighbourhoods contain collections of arcs hopelessly folded up.
3. Now, for arbitrary n , a glance at the derivative shows that.....

237 glue

1. Then M is obtained by glueing hgluing X to Y along Z .
2. There is a natural way to glue the associated varieties together along their common boundary.
3. The set A is obtained from B by removing a neighbourhood of C and gluing in a copy of D .

238 generalize ,[see also: extend]

1. Corollary 2 generalizes and strengthens Theorem 3 of [9]. This approach does not seem to generalize to arbitrary substructures.
2. It is not clear to what extent this can be generalized to other varieties of loops.

239 generally

1. However, M is generally not a manifold.
2. It is not generally possible to restrict f to the class D .
3. More generally this argument also applies to characterizing Hurewicz subsets of I .

240 generate

1. This is slightly at odds [= inconsistent] with the terminology of [4], as Fox defines the trace filter to be the normal filter generated by A .
2. The family of 4-sets will be used to generate a symmetry outside N but in M .

241 get

1. [see also: obtain, acquire, make, arrange, overcome, circumvent]
2. Apply Theorem 3 to get a function.....
3. We thus get $f = g$. [Not: “We thus get that $f = g$.”]
4. We do not expect to get F closed.
5. The induced topology is not compact, but we can always get it to be contained in a Bohr topology.
6. Finally, multiplication by a permutation matrix will get the exponents in descending order.
7. Fortunately, F does not get too close to p .
8. To get around this difficulty, assume that.....
9. It is also tempting to get round this problem by working with.....

242 futile, [see also: doomed]

Having established (1), one might be tempted to try to extend this result to general p through the choice of a suitable ideal B . Alas, as we shall see now, this attempt is futile.

243 future

1. For future reference, we record this in the following corollary.
2. We quote for future reference another result of Fox: there exists.....
3. This is an interesting area for future research.
4. For future use, choose any monotone $h(m)$ tending to infinity such that.....
5. These upper bounds are too large to be useful in computer calculations in general, but the ideas in the proofs will surely contribute to better bounds in the future.

244 gain

1. [see also: get, obtain, achieve] The theorem gains in interest if we realize that.....
2. Thus it is reasonable to attempt, using this homeomorphism, to gain an understanding of the structure of M .
3. It is useful to consider some rather simple examples to gain some intuition.
4. A change in perspective allows us to gain not only more general, but also finer results than in [12].

245 gap

1. The statement does appear in [3] but there is a simple **gap** in the sketch of proof supplied.
2. The theory of correspondences may be viewed as bridging the **gap** between.....

246 gather

1. [see also: collect, combine, piece together] We gather here various notation for future reference.
2. In this section we gather some miscellaneous results that are more or less standard.
3. The data were gathered for about a year.

247 general

1. His techniques work just as well for general v .
2. How are these two optimality notions related? In general, they are not.

3. While topological measures resemble Borel measures, they in general need not be sub-additive.
4. We suspect that more can be said in general about which subextensions can be obtained from finite-rank modules, but we will not explore this matter further here.

248 generality

1. There is no loss of generality in assuming that.....
2. This involves no loss of generality.
3. Without loss of generality we can assume that.....
4. [In many cases, the phrase “without loss of generality” can be omitted: write simply: We can clearly assume that..... Avoid using the abbreviation “w.l.o.g.”]
5. Without losing any generality, we could have restricted our definition of integration to integrals over all of X . [Not: “Without losing”]
6. It simplifies the argument, and causes no loss of generality, to assume.....
7. A completely different method was used to establish Theorem 2 in full generality.
8. Rather than discuss this in full generality, let us look at a particular situation of this kind.
9. A number of authors have considered, in varying degrees of generality, the problem of determining.....
10. It seems preferable, for clarity’s sake, not to present the construction at the outset in the greatest generality possible.
11. It seems that the relations between these concepts emerge most clearly when the setting is quite abstract, and this (rather than a desire for mere generality) motivates our approach to the subject.

249 first

1. [see also: original]
2. He was the first to propose a complete theory of triple intersections.
3. Because N. Wiener is recognized as the first to have constructed such a measure, the measure is often called the Wiener measure.
4. Let S_i be the first of the remaining S_j .
5. The first two are simpler than the third. [Or: the third one; not: “The first two ones”]

6. As a first step we shall bound A below.
7. Here is a first relation between $L(G)$ and endotrivial kG -modules.
8. We do this in the first section, which the reader may skip on a first reading.
9. At first glance, this appears to be a strange definition.
10. The first and third terms in (5) combine to give.....
11. the first author = the first-named author
12. [see also: initially, originally, beginning, firstly] First, we prove (2). [Not: "At first"]
13. We first prove a reduced form of the theorem.
14. Suppose first that.....
15. His method of proof was to first exhibit a map.....
16. In Lemma 6.1, the independence of F from V is surprising at first.
17. It might seem at first that the only obstacle is the fact that the group is not compact.
18. [Note the difference between first and at first: first refers to something that precedes everything else in a series, while at first [= initially] implies a contrast with what happens later.]

250 exception

1. There are few exceptions to this rule.
2. As the proof will show, these properties, with the exception of (c), also hold for complex measures.
3. With the exception noted below, we follow Stanley's presentation [3, Sec. 2].

251 eventually, [= in the end; 6= possibly]

1. The iterates eventually reach the value 1.
2. Then we can find some net (s_k) which eventually leaves every compact subset of G .
3. It will eventually appear that the results are much more satisfactory than one might expect.
4. an eventually increasing sequence

252 correspondence

1. Observe that A is thereby put into one-to-one **correspondence** with B .
2. The elements of C are in one-to-one **correspondence** with.....

253 converse

1. [of sth; to sth; see also: inverse, opposite, reverse]
2. In this section we investigate under what conditions the converse holds.
3. The converse is far from obvious.
4. For the converse, consider.....
5. This theorem is a converse hpartial conversei of Theorem 2.

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254 Appendix

Exercise o1 (0.25×8 marks): Re-write the following sentences in ordinary English.

- [wɪtʃ kʌm fɜːst ðə 'tʃɪkɪn ɔːr ðiː, eg]
- [tə'deɪ aɪ sə'dʒest ə ,sʌbstɪ'tjuːʃən ɒv sʌm 'kwɛstʃənz]

Exercise o2 (4 marks): Write in full form

$$|g.f| = g.f \quad \text{and} \quad \left(\frac{|g|}{\|g\|_q} \right)^q = \left(\frac{|f|}{\|f\|_p} \right)^p \quad a.e$$

and

$$e^A = \lim_{n \rightarrow \infty} \sum_{k=0}^n \frac{A^k}{k!}.$$

Exercise o3 (8 marks): *i*) Translate the following abstract in French language.

Abstract. Separable metric space possesses special properties that some of problem in analysis can be solved in these spaces easily. The question which appears basically is what properties are needed for metric spaces to be separable. We will answer to this question exactly in this article and even will get a special structure for separable metric spaces that are the same topological and algebraic structure of real numbers.

ii) Give the phonetic symbols of the previous abstract.

Exercise o4 (4 marks): Translate in English language.

a) Dans tous les exercices, on se place dans un espace métrique (E, d) . Les parties de E seront notées A, B , etc., les points de E seront notés x, y , etc.

b) **Suites.**

Exprimer la convergence d'une suite de points de E à l'aide des boules ouvertes de E . En déduire que toute sous-suite d'une suite convergente est convergente.

c) Un voisinage d'un point a est une partie de E contenant une boule ouverte centrée en a .

d) **Définition.** Un espace métrique est un ensemble E muni d'une fonction $d : E \times E \rightarrow \mathbb{R}_+$ vérifiant pour tout triplet $(x, y, z) \in E$, on a

$$\diamond) d(x, y) = 0 \iff x = y \quad (\text{positivité})$$

$$\diamond) d(x, y) = d(y, x) \quad (\text{symétrie})$$

$$\diamond) d(x, z) \leq d(x, y) + d(y, z) \quad (\text{inégalité triangulaire}).$$

Une telle fonction est appelée distance sur E .

Good Luck

Inner Product Space

In mathematics, a vector space or function space in which an operation for combining two vectors or functions (whose result is called an inner product) is defined and has certain properties. Such spaces, an essential tool of functional analysis and vector theory, allow analysis of classes of functions rather than individual functions. In mathematical analysis, an inner product space of particular importance is a Hilbert space, a generalization of ordinary space to an infinite number of dimensions.

A point in a Hilbert space can be represented as an infinite sequence of coordinates or as a vector with infinitely many components. The inner product of two such vectors is the sum of the products of corresponding coordinates. When such an inner product is zero, the vectors are said to be orthogonal. Hilbert spaces are an essential tool of mathematical physics.

Exercise 01 : (8 marks)

- 1) Give another title of the text. (1 mark).
- 2) Find a word or expression in the text which, in context, is similar in meaning to :
Farness, Series, unlimited, interior, boundless (2.5 marks)
- 3) Turn from passive into active the following sentence (2 marks)
- *A point in a Hilbert space can be represented as an infinite sequence of coordinates.*
- 4) Re-write the following words in ordinary English (2.5 marks)

- [ɪnɪ'kwɒlɪtɪ], [ɪ'nɪʃəl], [pɒlɪ'nəʊmɪəl], [ʌn'baʊndɪd]
- ['ældʒɪbrə], [ɪ'senʃəl], ['neglɪdʒəbl], [dɪfərənsɪ'eɪʃən],
['prɒpətɪ]

Exercise 02 (4 marks) : Translation

Translate the following paragraph in English language.

Théorème. Pour qu'un espace métrique (E,d) soit complet, il faut et il suffit que, toute suite décroissante de boules fermées de rayons tendant vers zéro admette une intersection non vide. (Plus précisément, cette intersection est réduite à un seul élément).

Exercise 03 (8 marks) : Vocabulary and Grammar

Choose the correct item.

- 1) Sarah is the prettiest girlour school. **then in of**
- 2) Paul's car is.....than Tom's. **fast fastest faster**
- 3) This dress is thein the shop. **more expensive most expensive expensive**
- 4) Bob the car at the moment. **washes is washing wash**
- 5) The sun in the west. **set is setting sets**
- 6) Peter is as.....as Sally . **clever cleverer cleverest**
- 7) Ann has two brothersof them are older than him. **none all both**
- 8) I was hungry so I madea sandwich . **me myself my**
- 9) She works in a bank,? **does she isn't she doesn't she**
- 10) That ispen. **Tom's Tom Toms'**
- 11) How would you feel if youyour car? **crash will crash crashed**
- 12) I.....read or write when I was four years old. **can't couldn't wasn't able**
- 13) Yoube rude to your parents. **must musn't coudn't**
- 14) I.....buy a new coat if I had enough money. **would must will**
- 15) He had studied hard so heanswer all the questions in the test.
was able to is able to can
- 16) She's known mea long time. **ago for since**

Ex 01 (0.5×17 marks): Complete the following notions by using the correspondent mathematical words and phrases.

1. Matrices with the property $A^*A = AA^*$ are said to be
2. Let $A \in M_n(\mathbb{R})$ be a square matrix. If $\det(A) \neq 0$, then A is
3. For square matrices A , the number $\rho(A) = \max_{\lambda \in \sigma(A)} |\lambda|$ is called the of the matrix A .
4. Let $A \in M_n(\mathbb{R})$ be a square matrix. The for A is to find solutions to the matrix equation $Ax = \lambda x$, where $\lambda \in \mathbb{K}$ and $x \in \mathbb{K}^n$ such that $x \neq 0$.
5. Recall that a function f is called an function if $f(-x) = f(x)$. Similarly, f is called an function if $f(-x) = -f(x)$.
6. A collection of vectors $\{v_1, v_2, \dots, v_k\}$ is said to be an set if $\langle v_i, v_j \rangle = 0$ for all $i \neq j$. If, in addition, $\|v_i\| = 1$ for all $i = 1, 2, \dots, k$, the set is called
7. A symmetric matrix $A \in M_n(\mathbb{R})$ is called if

$$x^t Ax > 0 \quad \text{for all } x \in \mathbb{R}^n.$$

8. A function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is called a function if for all $\theta \in [0, 1]$, and for any $x, y \in \mathbb{R}^n$ we have

$$f(\theta x + (1 - \theta)y) \leq \theta f(x) + (1 - \theta)f(y).$$

9. The inequality : $\|x + y\| \leq \|x\| + \|y\|$ which is known ""
10. A set A is said to be iff A is contained in the range of some sequence (briefly, the elements of A can be put in a sequence).
11. In any metric space (E, d) , every compact subset K is
12. (.....). If $f : [a; b] \rightarrow \mathbb{R}$ is continuous and $f(a) < y < f(b)$, then y must be a value of f .
13. A topological space (X, τ) is T_2 (or Hausdorff) iff given $x \neq y$ in X , there exist of x and y .
14. The closed graph theorem states the following: If X is a space and Y is a space, then the graph of a linear map T from X to Y is closed if and only if T is continuous.

Ex 02 (3.5 marks). Write the following formulas in full form :

$$\sqrt[n]{n! + x} \leq 1 \quad i, e., \quad \sum_{i=1}^n \frac{e^{x_i^2 + \cos(x_i^n)}}{\sqrt{\sin^3(x_i) - 1}} \quad \frac{\pi - \varphi + \xi}{(\varepsilon + \rho + \phi)^\omega} \quad f^{(n)} \notin A \quad \frac{\partial f}{\partial x} \quad A \cap B = \emptyset$$

Ex 03 (5.5 marks). Read carefully the following text.

The absolute value function on \mathbb{R} and the modulus on \mathbb{C} are denoted by $|\cdot|$ and each gives a notion of length or distance in the corresponding space and permits the discussion of convergence of sequences in that space or continuity of functions on that space. In this work, we shall extend these concepts to a general linear space E . A seminorm on the linear space E is a function $p : E \rightarrow \mathbb{R}$ for which $p(\alpha x) = |\alpha|p(x)$ and $p(x + y) \leq p(x) + p(y)$ for all $\alpha \in \mathbb{K}$ and $x, y \in E$. The pair (E, p) is called a seminormed space. We study some properties concerning seminormed spaces, for example, a closed subspace of a seminormed space is complete but the reciprocal is false. Finally, we prove that a complete subspace of a normed space is closed.

1) Give a suitable title of the text.

2) Find a word or expression in the text which, in context, is similar in meaning to :
converse, full, series, notion, map (mapping), to expand.

3) Turn from active to passive the following statements:

a- We had studied some properties concerning seminormed spaces.

b- In this work, we shall extend these concepts to a general linear space E .

c- Hilbert let many problems without proof.

d- We may use the contraction mapping theorem to prove the existence and uniqueness of solutions.

4) Give the phonetic of the following words.

discussion, normed, general, called, value, properties.

Ex 04 (3 marks): Give one word (if it is possible in relation with mathematics) for every symbol.

ə	i:	ɪ	æ	e	ʌ	
ɔ:	ɒ	a:	u:	ʊ	ə:	
eɪ	eə	aɪ	ɔɪ	aʊ	ɪə	əʊ

Good luck.