Part I:
Basic Elements

Chapter 2:
System number

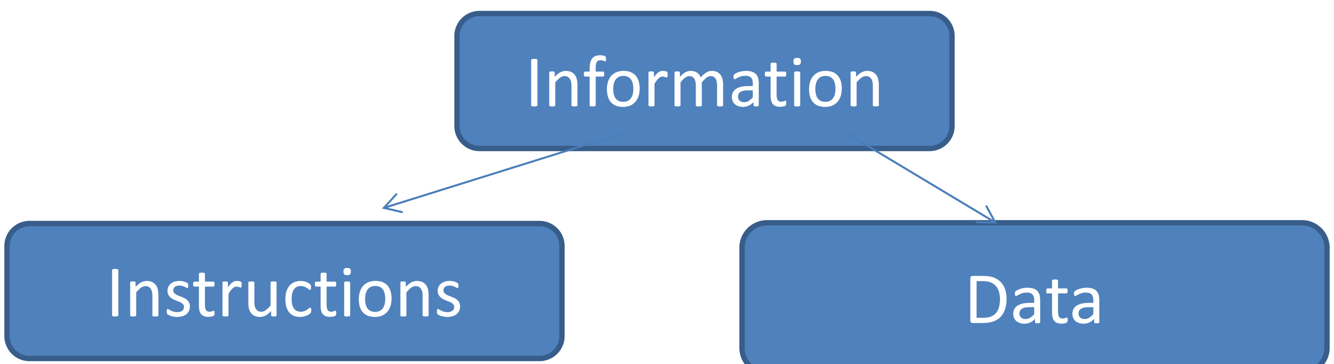
2024-2025

Representation of the information

The computer can process various types of information:
Numerical values, Texts, Images, Sound, …

BUT

All this information is stored in digital form



Coding of information

Whatever its nature (number, text, image, sound, or video), digital information processed by a computer is always represented in binary form **(a sequence of 0 and 1). For example: 01111011, 11000000…..**

- The smallest unit of information transmitted by a computer is called Bit (BInary digiT) (which can take two values: 0 or 1)
- A unit of information made up of 8 bits is called a “byte”

Some sizes

- 2^{10} bits = 1024 bits = 1 **Kb** (1 Kilo bits) / 2^{10} Ø = 1024 Ø = 1 **KØ** (1 Kilo Ø)

- 2^{10} Kb = 1024 Kb = 1 **Mb** (1 Mega bits) / 2^{10} KØ = 1024 KØ = 1 **MØ** (1 Mega Ø)

- 2^{10} Mb = 1024 Mb = 1 **Gb** (1 Giga bits) / 2^{10} MØ = 1024 MØ = 1 **GØ** (1 Giga Ø)

- 2^{10} Gb = 1024 Gb = 1 **Tb** (1 Tera bits)/ 2^{10} GØ = 1024 GØ = 1 **TØ** (1 Tera Ø)

Definition of Information Coding

The coding of information consists of establishing a correspondence between the (usual) external representation of the information (text, number, image, etc.), and its internal representation in the machine, which is always a series of bits.

Example: the number 22

- Its external representation = 22
- Its internal representation (in binary) = 00010110

How to represent numbers (integers, real, etc.) and characters (letters, mathematical symbols, etc.) in the machine?

Steps in coding information

The coding of information passes by three steps:

1. Representation of information by a series of numbers (Digitalization)
2. Encoding each number in a binary form
3. Represent each binary element by a physical state (electrical signal)

Representation of numbers: Number systems

- **Number systems describe how numbers are represented.**
- A number system is defined by:
  - **Alphabet (A):** A set of symbols (numbers): $A=\{a_1, a_2, \dots, a_n\}$
  - Rules for writing numbers: Juxtaposition of symbols
  - $\mathbf{a_1 a_3}$**:** is a word
- In a number system, the number of distinct symbols is called the base of the number system (the cardinal of the set A).
- In computing, the most used bases are **binary, octal, and hexadecimal.**

Representation of numbers in a base b

- A number $(XXX)_b$ indicates the representation of a number XXX in the base b.
- The usual bases that we know and use every day are:
  - **base 10 (decimal system)** to represent different quantities, different figures and numbers, and
  - **base 60** to represent time.

How to represent a number in a base b?

If $b \leq 10$, we simply use the numbers 0 to b-1

Example: base 8 (octal system): any number will be the combination of digits belonging to the set {0,…, 7}

Representation of numbers in a base b (continued)

If b > 10, we simply use the numbers 0 to 9 then the letters in alphabetical order.

Example:

Base 16 (hexadecimal system): any number will be the combination of symbols belonging to {0,…, 9, A, B, C, D, E, F} such that: (A=10 , ….., F=15).

A number of n digits (symbols) is a sequence (a_i), $0 \leq i \leq n-1$: $a_{n-1} \ldots\ldots a_1\ a_0$ such that: a_0 is the least significant term and a_{n-1} is the most significant term.

Decimal system

- It is the number system that we frequently use in our daily activities.
- Based on 10 symbols {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} === base 10
- It is a **positional system**: Each position has a weight.

Example:

The number **$\underline{5}36\underline{8}$ is written as following** :

$$\mathbf{5368} = 8*10^0+6*10^1+3*10^2+5*10^3$$

Low weight

Heavy weight

$$5368,135=5*10^3+3*10^2+6*10^1+8*10^0+1*10^{-1}+3*10^{-2}+5*10^{-3}$$

Integer part

Decimal part

Binary system

- Base (b)=2
- The system used in computers
- It uses two digits {0,1}

- Example: $\mathbf{(10111101)_2}$

- ***The polynomial form :***

$$(\mathbf{10111101})_2 = 2^0*1+2^1*0+2^2*1+2^3*1+2^4*1+2^5*1+2^6*0+2^7*1$$

$$= (189)_{10}$$

Transcoding: Bases change

Transcoding (or bases conversion) is the operation, which allows to go from the representation of a number in one base to its representation in another base.

Decimal → Base b

Number N= integer part, decimal part (example: 15, 23)

Integer part: The successive division method

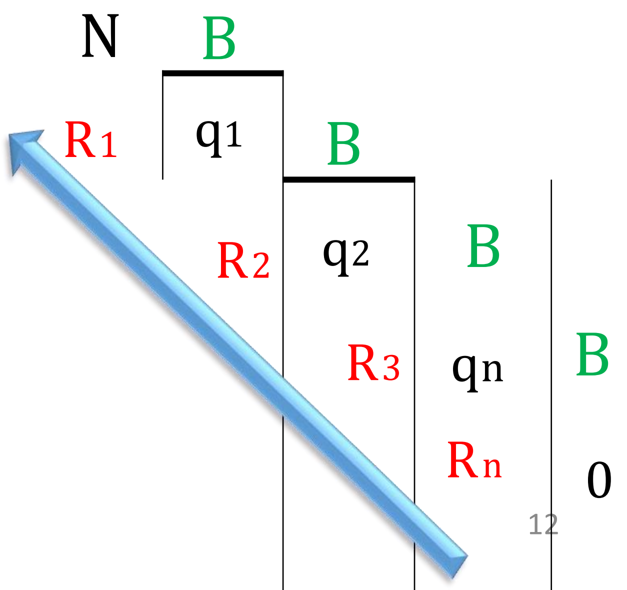
Divide the number by B

Then the quotient by B

and so on until obtaining a zero quotient

take the remainders of successive divisions on the base X in the opposite direction.

$(N)_{10}=(R_n..R_3R_2R_1)_B$



Decimal base to base b

- **Integer part:**

  Successive multiplications until having a zero result or obtaining a given precision

Example:

$(115{,}23)_{10} = (?)_2$

With a precision of 6 places after the decimal point.

We treat each part separately

Integral part:

115÷2=57 remainder 1

57÷2= 28 remainder 1

28÷2= 14 remaider 0

14÷2= 7 remainder 0

7÷2= 3 remainder 1

3÷2=1 remainder 1

1÷2=0 remainder 1 (quotient=0 stop)

Hence: $(115)_{10} = (1110011)_2$

$(0,23)_{10}=(?)_2$

- Now let's move on to the decimal part :

  $(0,23)_{10}=(?)_2$

  0,23x2= <u>**0**</u>,46 integer number is 0

  0,46x2= <u>**0**</u>,92 integer number is 0

  0,92x2= <u>**1**</u>,84 integer number is 1

  0,84x2= <u>**1**</u>,68 integer number is 1

  0,68x2= <u>**1**</u>,36 integer number is 1

  0,36x2= <u>**0**</u>,72 integer number is 0

- **Hence: $(0,23)_{10}$=(0,001110)2**
- Final result: $(115,23)_{10}$= (**1110011, 001110**)$_2$

Decimal Base to the Octal and Hexadecimal base

Decimal → Octal

$$\begin{array}{r|l}
25 & \underline{8} \\
1 & 3 \;\; \begin{array}{|l} \underline{8} \\ 0 \end{array} \\
 & 3
\end{array}$$

$(25)_{10} = (31)_8$

Decimal → hexadecimal

$$\begin{array}{r|l}
25 & \underline{16} \\
9 & 1 \;\; \begin{array}{|l} \underline{16} \\ 0 \end{array} \\
 & 1
\end{array}$$

$(25)10 = (19)_{16}$

Conversion from base b to the decimal base

- Use polynomial expansion

$X = (a_{n..}a_2a_1a_0)_b$

$= b^0a_0 + b^1a_{1+\dots}b^na_n = (\sum a_ib^i)_{10}$

Examples:

- $(11011101{,}1)_2 = 2^{-1}*1 + 2^0*1 + 2^1*0 + 2^2*1 + 2^3*1 + 2^4*1 + 2^5*0 + 2^6*1 + 2^7*1 = (221{,}5)_{10}$
- $(175{,}26)_8 = 8^{-1}*2 + 8^{-2}*6 + 8^0*5 + 8^1*7 + 8^2*1$
- $(14)_{16} = 16^0*4 + 16^1*1 = (20)_{10}$
- $(1011)_2 = (1 \times 2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0)_{10} = (1\times8 + 0\times4 + 1\times2 + 1\times1)_{10} = (11)_{10}$
- $(16257)_8 = 1 \times 8^4 + 6\times8^3 + 2\times8^2 + 5\times8^1 + 7\times8^0 = 1\times4096 + 6\times512 + 2\times64 + 5\times8 + 7$

  $= 4096 + 3072 + 128 + 40 + 7 = 7343$
- $(F53)_{16} = 15 \times 16^2 + 5 \times 16^1 + 3 \times 16^0 = 15 \times 256 + 5 \times 16 + 3 = 3840 + 80 + 3 = 3923$

Conversion from binary base to the octal base

- Making 3-bits groups starting from the least significant one.
- Replace each group with the corresponding octal value.
- 3 binary digits ⇒ one octal digit

$$(\underbrace{10}_{2}\underbrace{111}_{7}\underbrace{101}_{5})_2$$

$$=$$

$$(275)_8$$

Conversion from the octal base to the binary base

- Replace each symbol in the octal base with its 3-bit binary value

Example: $(213)_8$

2 1 3

010 001 011

Arithmetic operations in Binary system: The addition

The addition

+	0	1
0	0	1
1	1	1 0

retain

$$\begin{array}{r} {\color{red}{11\ 1}} \\ 11011 \\ +\quad \\ 10110 \\ \hline (110001)_2 \end{array}$$

$$\begin{array}{r} {\color{red}{1\,1\,1}} \\ 10111 \\ +\quad \\ 10111 \\ \hline (101110)_2 \end{array}$$

Arithmetic operations : The subtraction

In binary

The substruction

-	0	1
0	0	1
1	1	0

Borrow 1

$$
\begin{array}{r}
{\scriptstyle 1} \\
11011 \\
- \quad {\scriptstyle 1} \\
10110 \\
\hline
(00101)_2
\end{array}
$$

Arithmetic operations : The multiplication

In binary

The multiplication

*	0	1
0	0	0
1	0	1

```
     11011
   *
       110
   1 1
     00000
+ 1 11011
+1 11011
  10100010
```

Arithmetic operations : The Division

In binary

The division

/	0	1
0	Meaningless	0
1	Meaningless	1

```
  11011 | 110
- 110   |------
  000   | 100
        |
     11 |
```

Application exercises

Perform the following operations and transform the result to decimal each time:

- $(1111,101)_2+(10,1)_2=(?)_2$
- $(45)_8+(75)_8=(?)_8$
- $(AB4)_{16}+(253)_{16}=(?)_{16}$