University of May 8, 1945 - Guelma
Faculty of Mathematics and Computer Science
Department of Computer Science
2nd Year Bachelor's Degree - Computer Science
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Exam: Numerical Methods (Duration: 2 hours)

Questions: (5 pts)

Choose the correct answer (only one):

1. The Jacobi method is:
   - A. A direct method for solving linear systems.
   - B. An iterative method for solving linear systems.
   - C. A method for calculating the determinant of a matrix.
2. The eigenvectors of a matrix A associated with an eigenvalue λ are obtained by solving the equation:
   - A. $(A - \lambda I)x = 0$
   - B. $(A + \lambda I)x = 0$
   - C. $(A - \lambda I)x = b$
3. The convergence of the Jacobi method is guaranteed if:
   - A. The matrix A is strictly diagonally dominant.
   - B. The matrix A is symmetric.
   - C. The matrix A is triangular.
4. In the IEEE-754 format, double precision (64 bits) uses:
   - A. 11 bits for the exponent and 52 bits for the mantissa.
   - B. 8 bits for the exponent and 23 bits for the mantissa.
   - C. 10 bits for the exponent and 21 bits for the mantissa.
5. A vector norm is zero if and only if:
   - A. Only one element of the vector is zero.
   - B. All elements of the vector are zero.
   - C. The sum of the elements of the vector is zero.

Exercise 1: (4 pts)

In this exercise, we will use the **binary8** format to encode (and decode) floating-point numbers. This format uses:

- 1 bit for the sign,
- 4 bits for the exponent,
- 3 bits for the mantissa (+1 normalization bit),
- with an exponent bias X = 7.

1. **Encode** the following decimal numbers in the **binary8** format (round to the nearest):
   - A = 12.75
   - B = -1.25

2. **Decode** the following numbers encoded in the **binary8** format (give the result in decimal):

   - C = $(01101011)_{b8}$
   - D = $(11010101)_{b8}$

3. **Perform** the following operations in the **binary8** format (round to the nearest):

   - A $\otimes$ B
   - C $\oplus$ D

Exercise 2: (6 pts)

Consider the following linear system written in equation form:

$$\begin{cases} -2x_1 - 2x_2 + 2x_3 - 2x_4 = -6 \\ -2x_1 - 2x_2 + x_3 = 1 \\ 6x_1 + 7x_2 - 8x_3 + 5x_4 = 11 \\ -6x_1 - 8x_2 + 9x_3 = 11 \end{cases}$$

1. **Write** the system in its matrix form (provide the matrix A and the vector b).
2. **Use** the Gaussian elimination method with partial pivoting to transform the system into an equivalent triangular system (swap with any row if the pivot is zero).
3. **Calculate** the solution of the obtained system.
4. **Verify** that the solution is valid for the original system.

Exercise 3 - MI: (5 pts)

The following recursive function is supposed to calculate the determinant of a square matrix A using the cofactor expansion along the first row. However, the code contains some errors.

```
function d = determinant(A)
    [n, m] = length(A);
    if n ~= m
        error('The matrix must be square');
    end
    if n == 1 % Base case
        d = A(1, 1) * A(2, 2) - A(1, 2) * A(2, 1);
    else      % Recurrence relation
        for j = 1:n
            A[i, j] = [];
            d = d + (-1)^(1+j) * A(1, j) * determinant(A);
        end
    end
end
```

- **Find**, **explain**, and **correct** each error so that the function works correctly.